

MATEMATIKA

Trigonometrijske funkcije

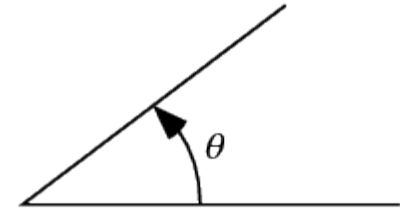
Trigonometrijske funkcije

Knjiga „*Matematika za IT*”

- Poglavlje „Trigonometrijske funkcije”, str. 51. – 68.

Mjera kuta

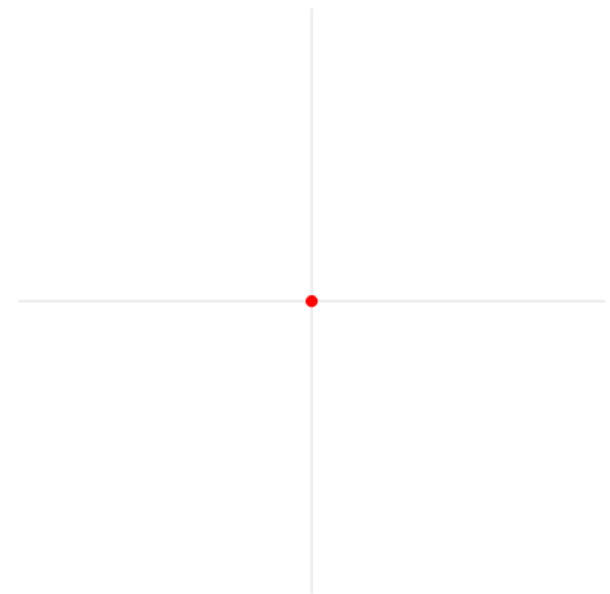
Kut je dio ravnine omeđen s dva polupravca koji imaju zajedničko hvatište.



Kut označavamo malim grčim slovima:
 $\alpha, \beta, \gamma, \delta, \theta, \phi \dots$

Mjerimo ga stupnjevima ili radijanima.

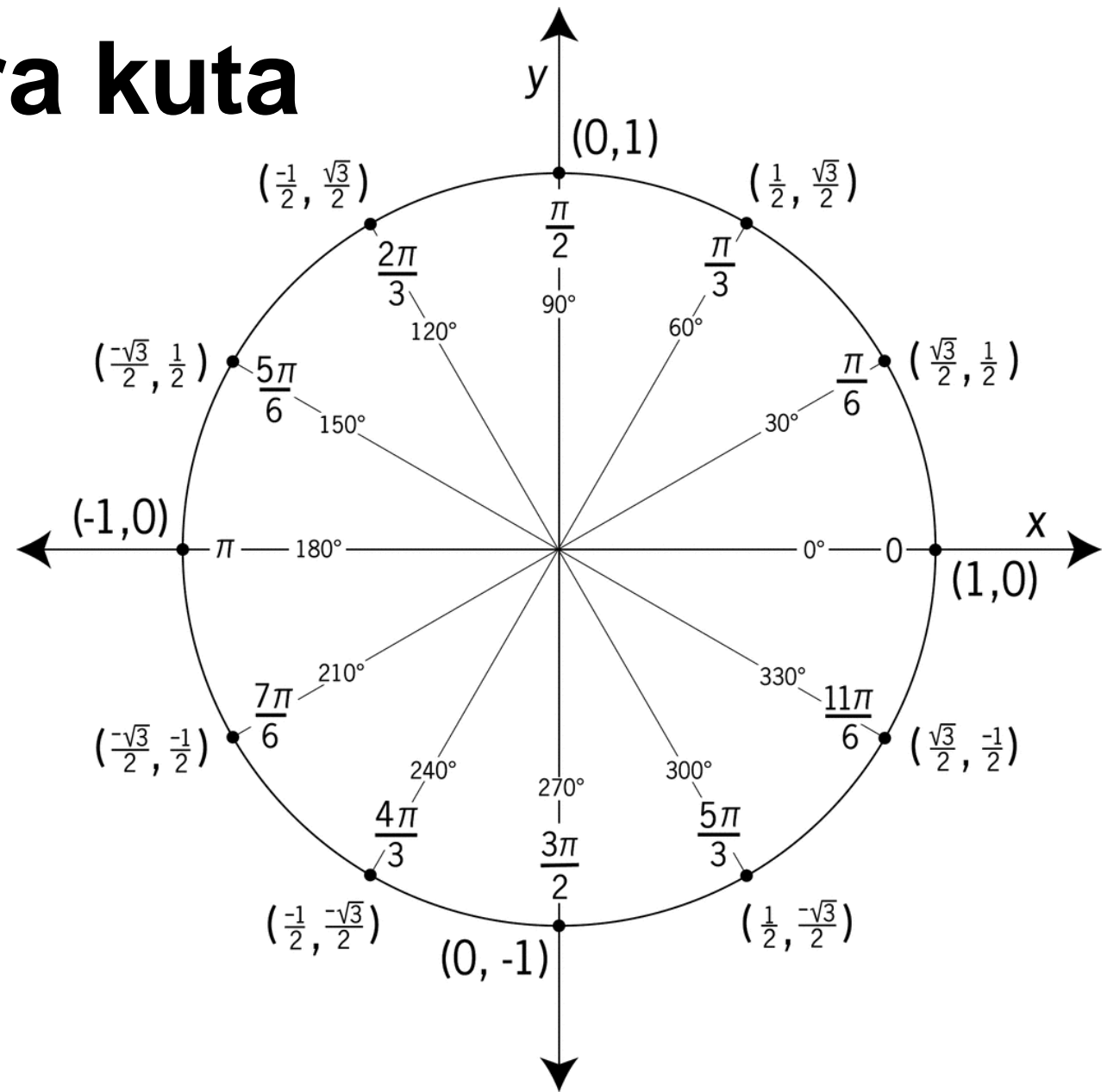
Puni krug označavamo mjerom od 360° , ili 2π radijana.



Mjera kuta

Veza između mjere kuta zapisane u stupnjevima α_s i mjere kuta zapisane u radjanima α_r :

$$\frac{\alpha_s}{180^\circ} = \frac{\alpha_r}{\pi}$$

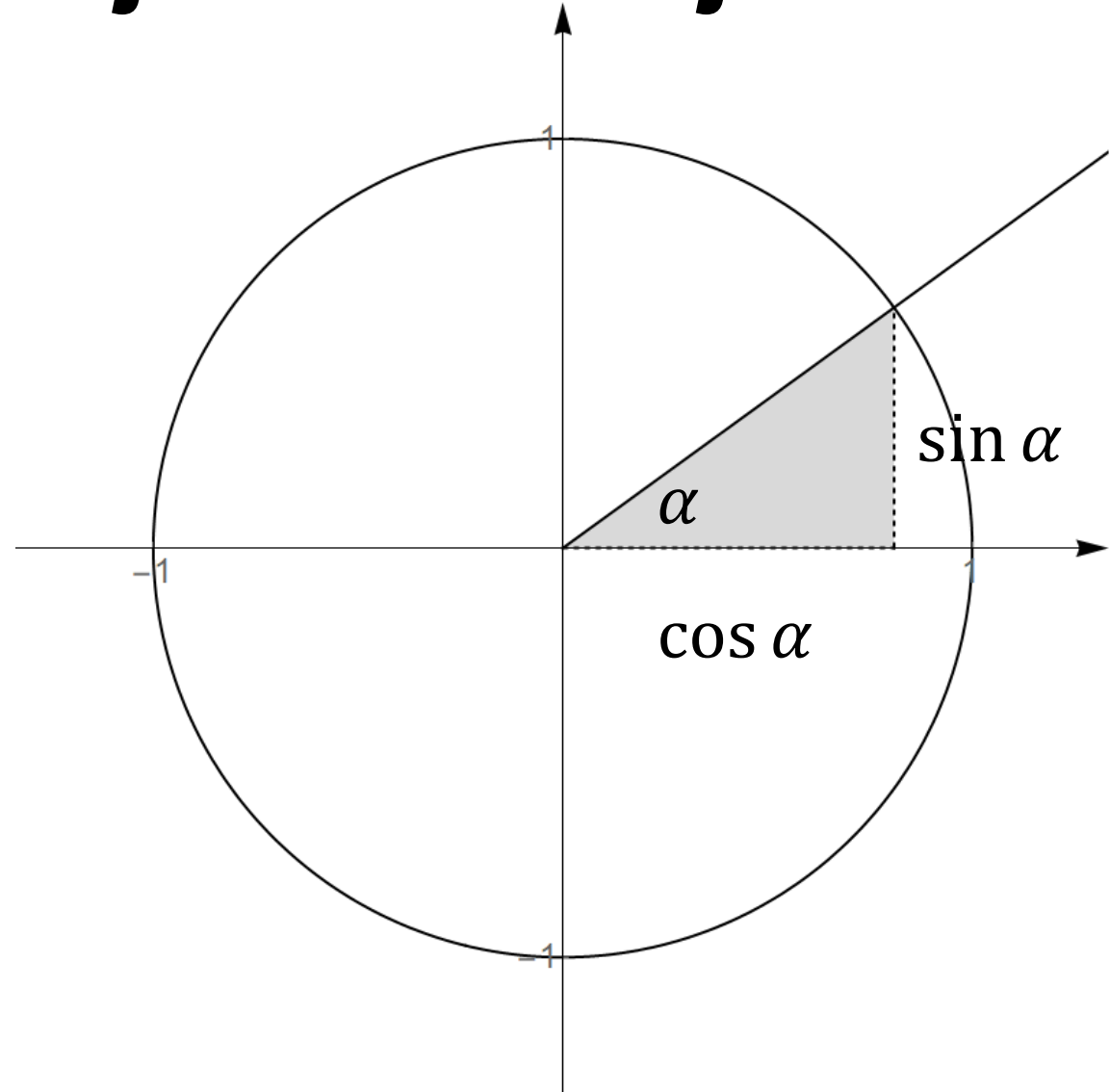


Trigonometrijske funkcije

Trigonometrijske funkcije čitamo na jediničnoj kružnici.

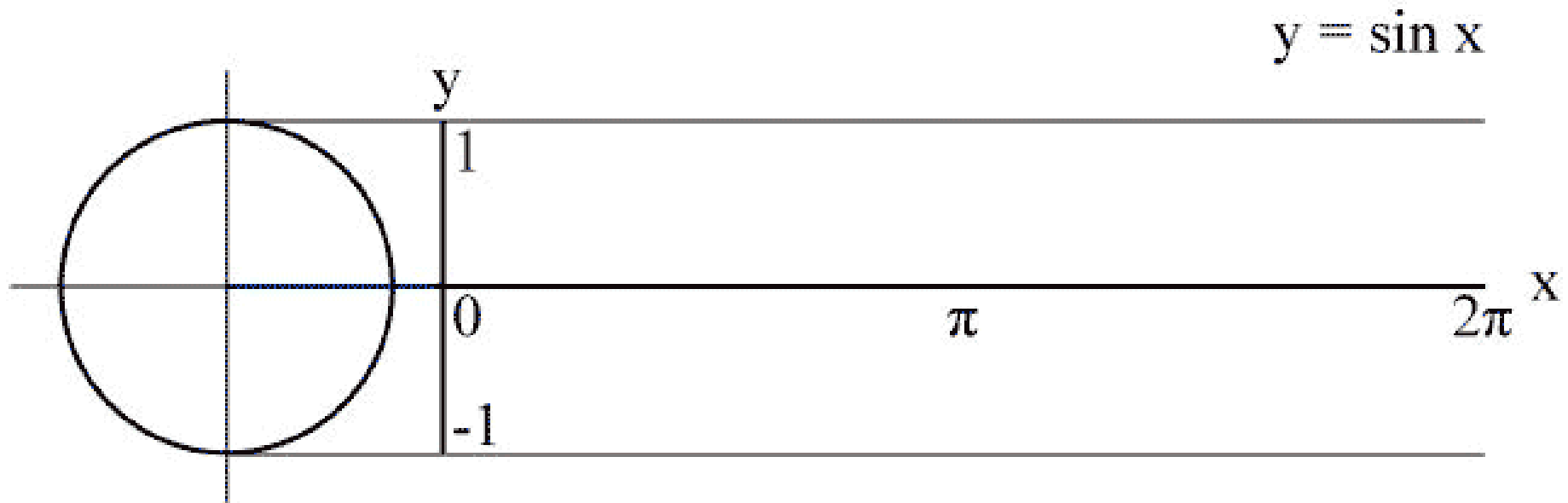
Sinus kuta je y -koordinata presjedišta kuta i jedinične kružnice.

Kosinus kuta je x -koordinata presjedišta kuta i jedinične kružnice.



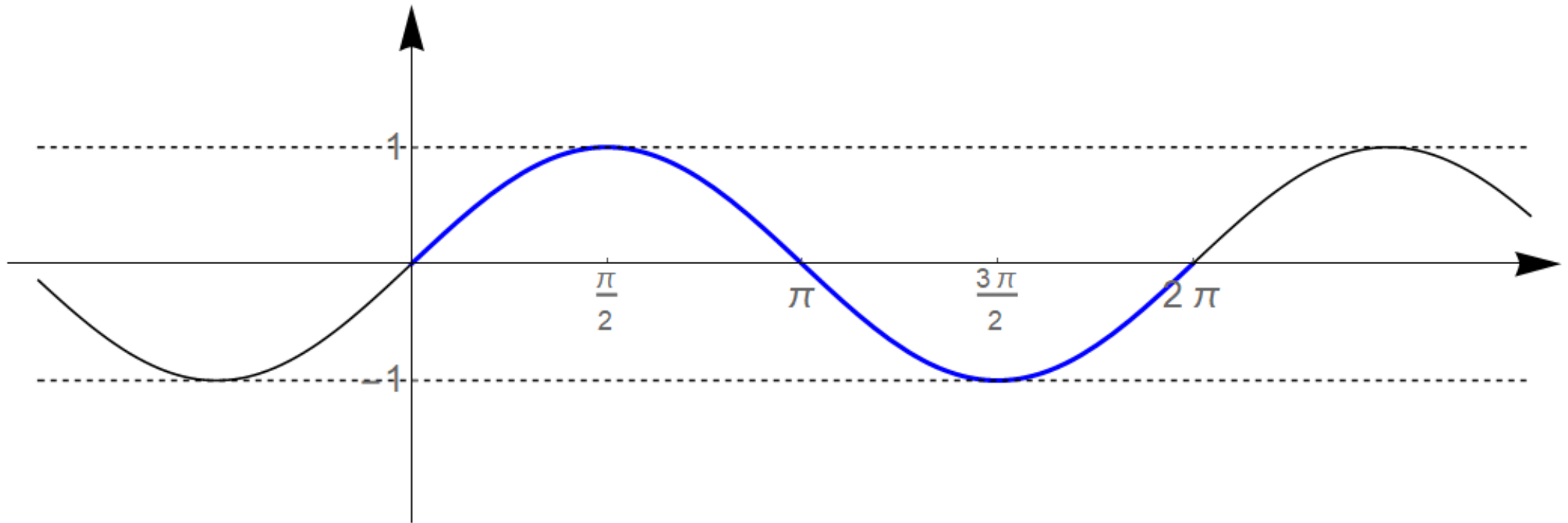
Trigonometrijske funkcije

Nakon definicije trigonometrijskih veličina (sinusa i kosinusa), promatramo kako te veličine ovise o promjeni kuta (kojeg označavamo kao varijablu, s x).



Trigonometrijske funkcije

Prikažimo grafički funkciju $f(x) = \sin x$.

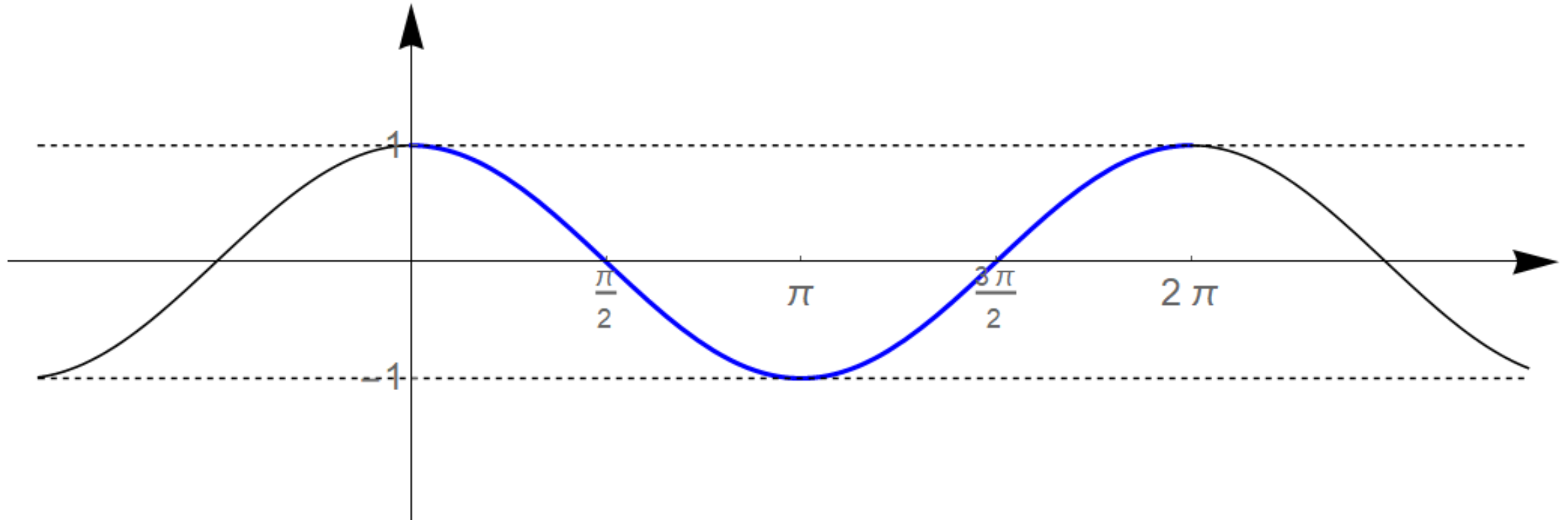


Domena: $D_f = \mathbb{R}$

Nultočke: $x = k \cdot \pi, \quad k \in \mathbb{Z}$

Trigonometrijske funkcije

Prikažimo grafički funkciju $f(x) = \cos x$.



Domena: $D_f = \mathbb{R}$

Nultočke: $x = \frac{\pi}{2} + k \cdot \pi, \quad k \in \mathbb{Z}$

Trigonometrijske funkcije

Tangens kuta definiramo:

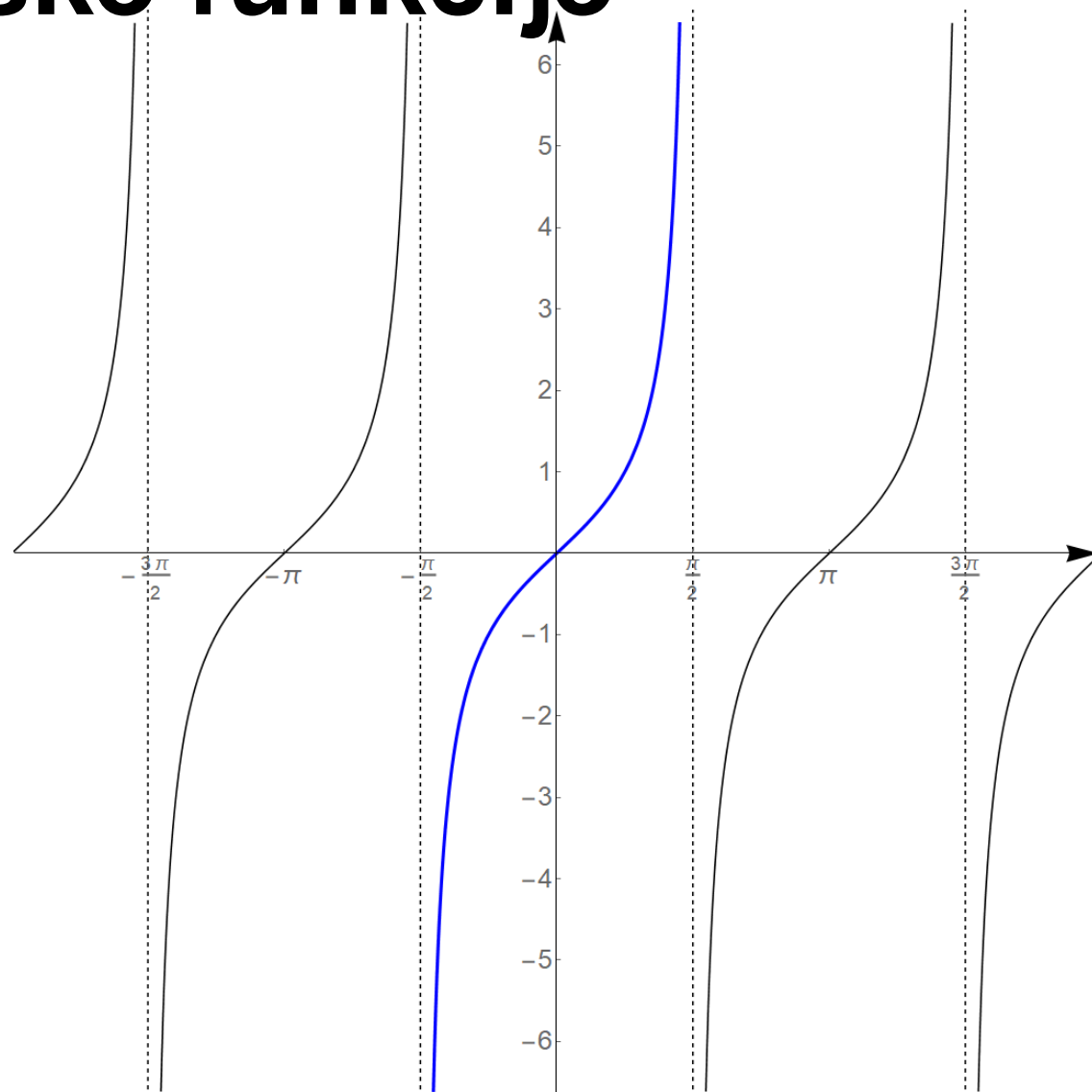
$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$

Domena funkcije $f(x) = \operatorname{tg} x$

$$D_f = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k \cdot \pi \right\}, \quad k \in \mathbb{Z}$$

Nultočke:

$$x = k \cdot \pi, \quad k \in \mathbb{Z}$$



Trigonometrijske funkcije

Kotangens kuta definiramo:

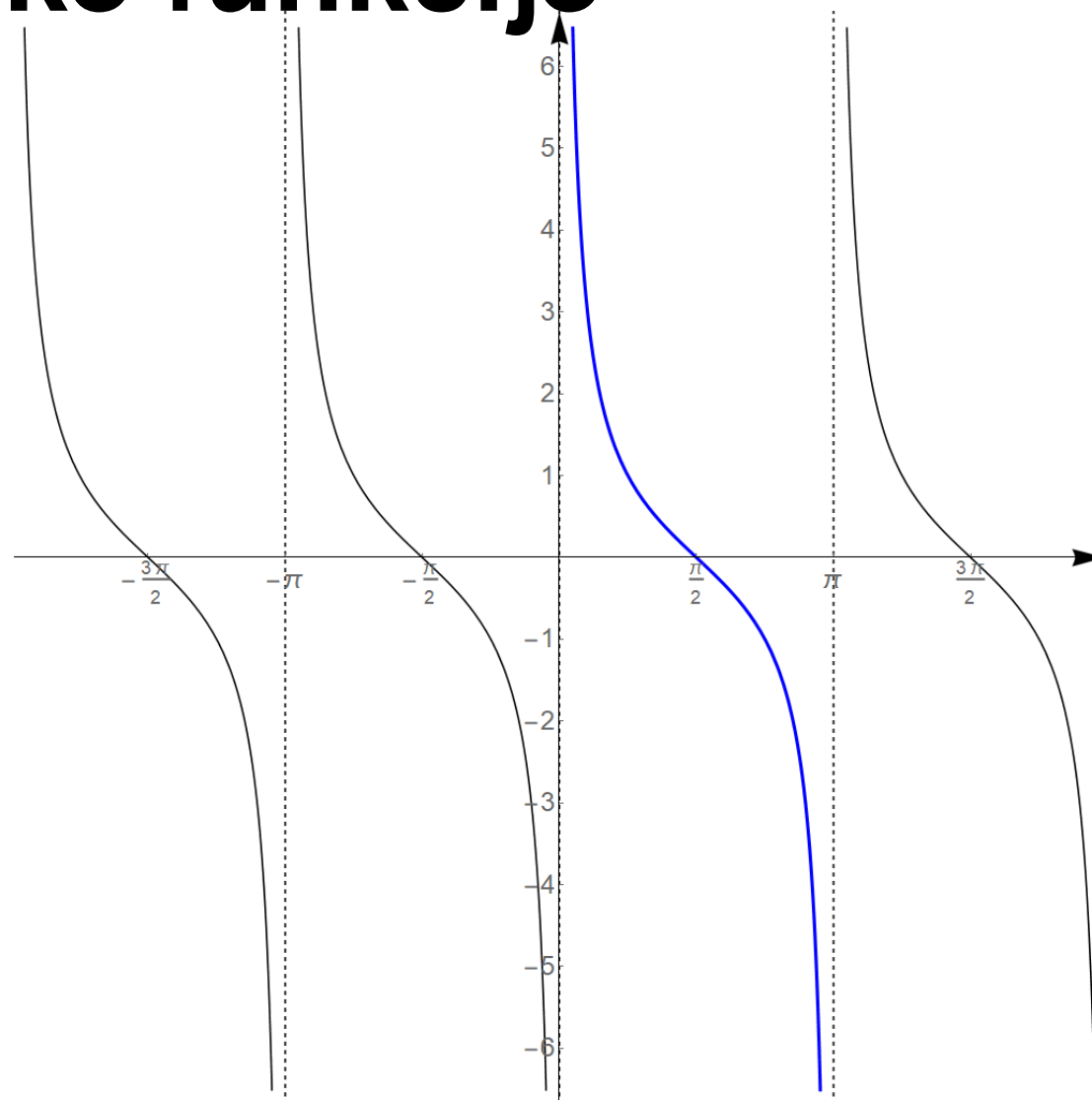
$$\operatorname{ctg} x = \frac{\cos x}{\sin x}$$

Domena funkcije $f(x) = \operatorname{ctg} x$

$$D_f = \mathbb{R} \setminus \{k \cdot \pi\}, \quad k \in \mathbb{Z}$$

Nultočke:

$$x = \frac{\pi}{2} + k \cdot \pi, \quad k \in \mathbb{Z}$$



Trigonometrijske funkcije

Geogebra:

<https://www.geogebra.org/m/b99RmZsp>

Skicirajte graf funkcije:

1. $y = 2 \sin x$

4. $y = -\sin x$

2. $y = \sin 2x$

5. $y = -\cos x$

3. $y = 1 + \sin x$

6. $y = \cos x - 1$

Osnovni identiteti

$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\operatorname{ctg} x = \frac{\cos x}{\sin x}$$

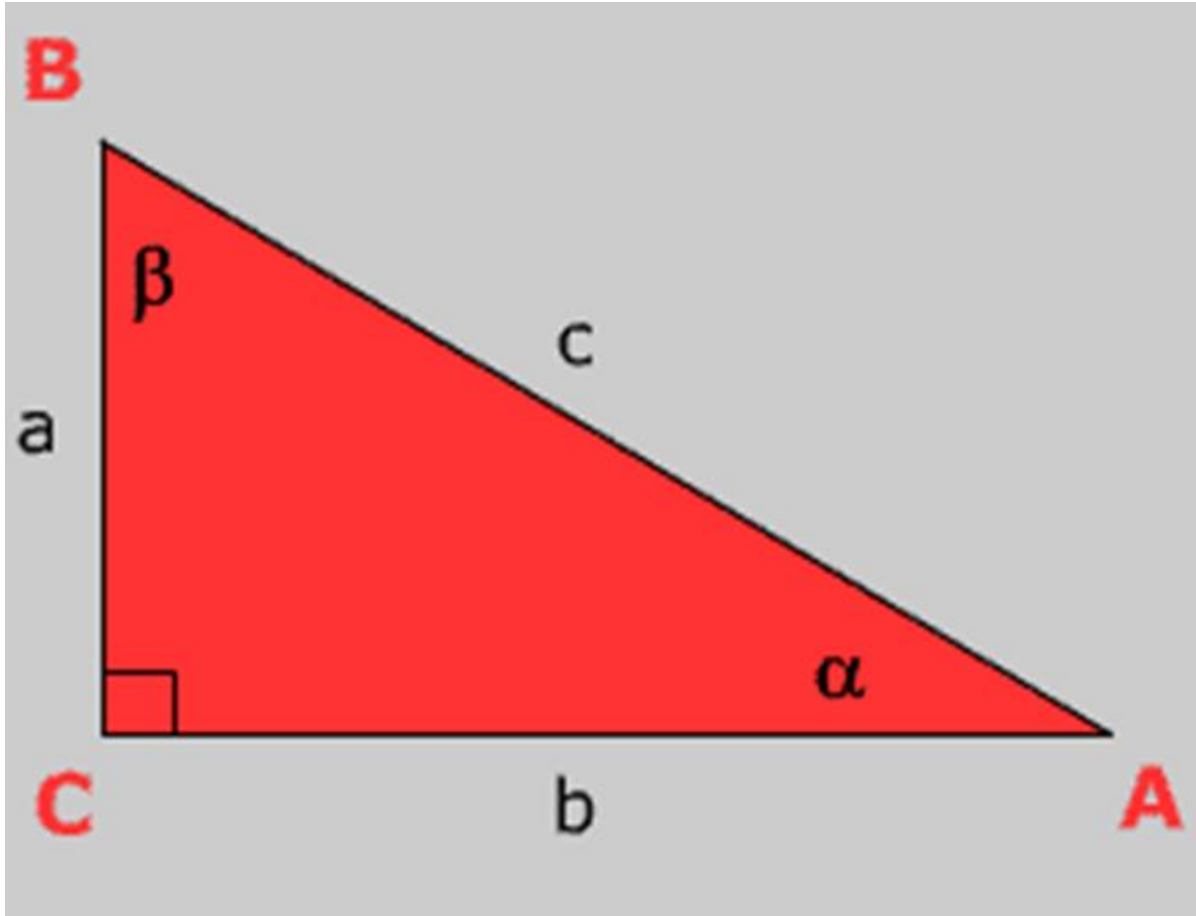
$$\sin 2x = 2 \sin x \cos x$$

$$\operatorname{tg} x = \frac{1}{\operatorname{ctg} x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

	0°	30°	45°	60°	90°	180°	270°	360°
α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\operatorname{tg} \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	∞	0
$\operatorname{ctg} \alpha$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	∞	0	∞

Pravokutni trokut

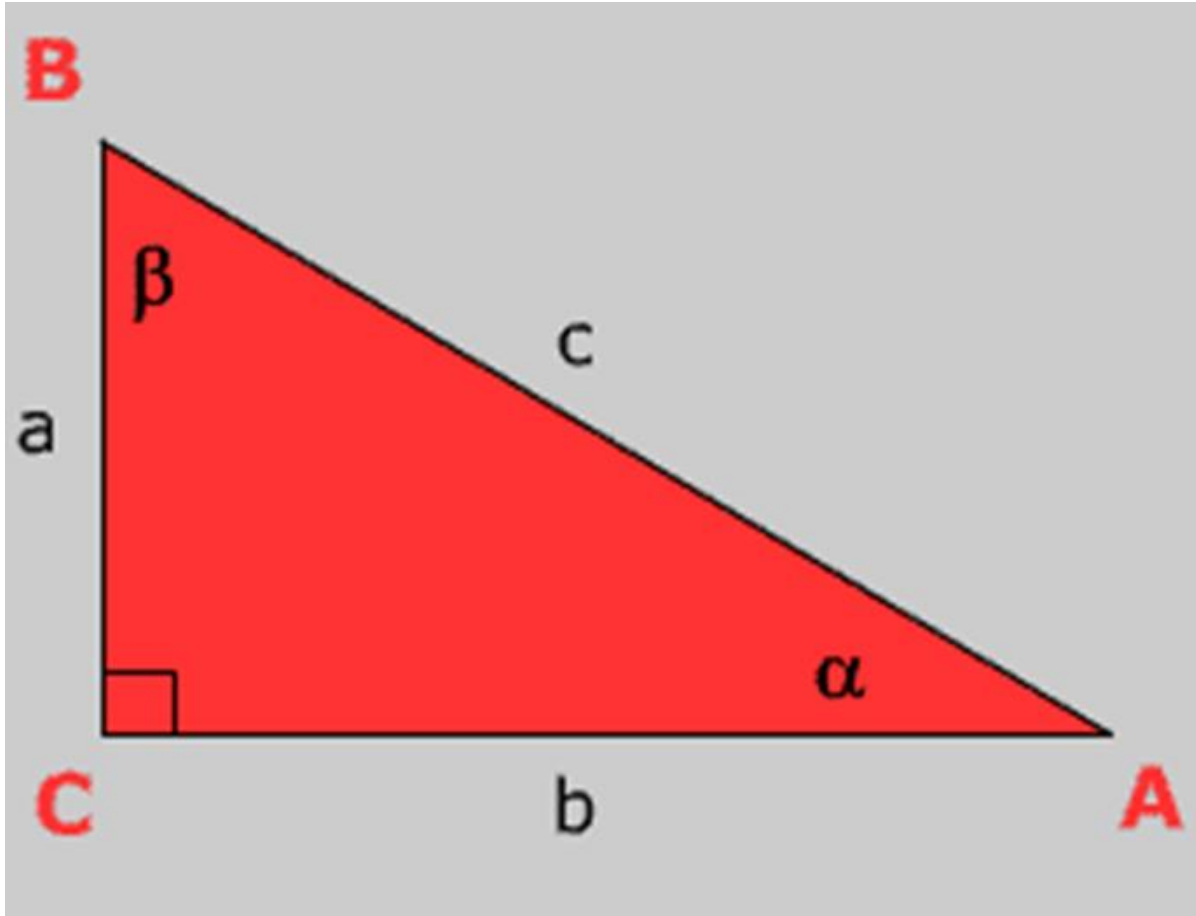


$$\alpha + \beta = 90^\circ$$

a, b - katete

c - hipotenuza

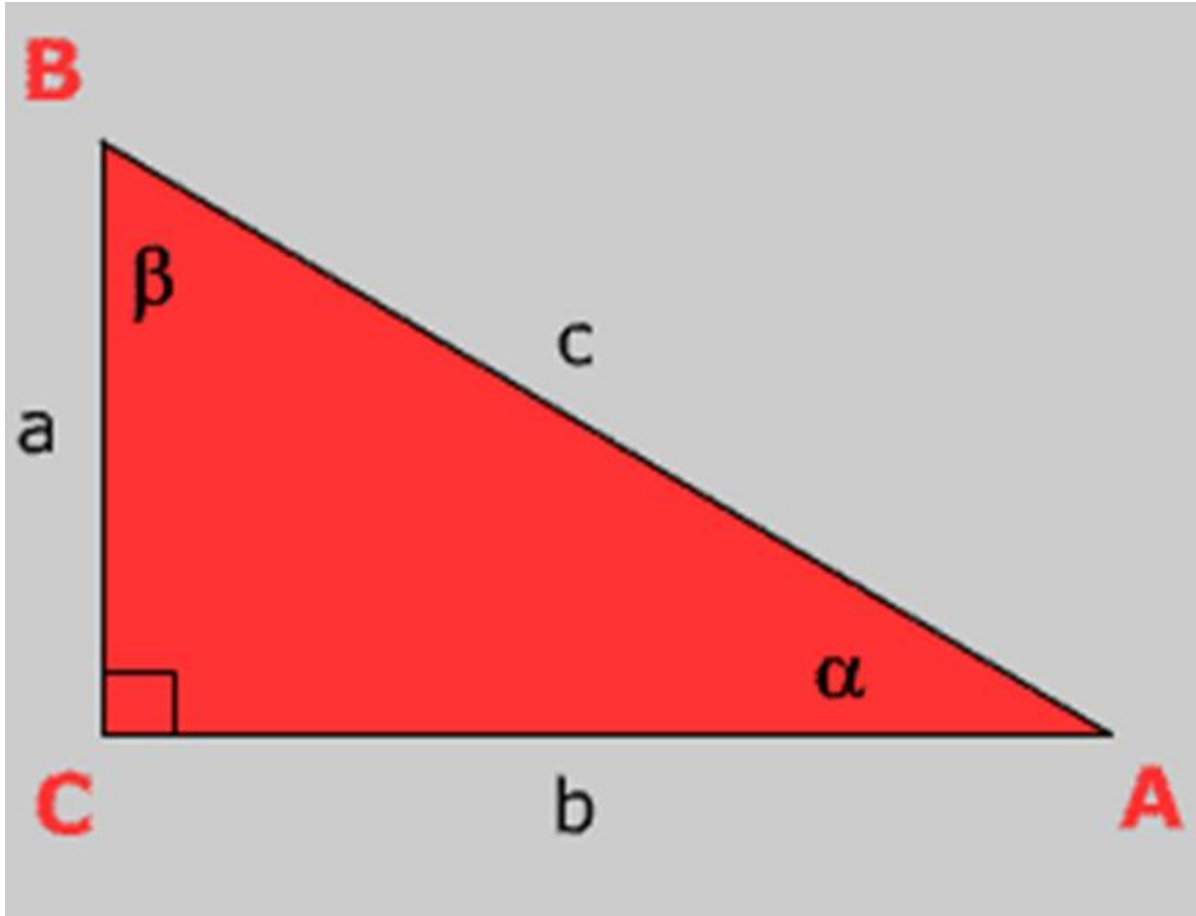
Pravokutni trokut



$$\sin \alpha = \frac{\text{nasuptotna kateta}}{\text{hipotenuza}}$$

$$\sin \alpha = \frac{a}{c} \qquad \sin \beta = \frac{b}{c}$$

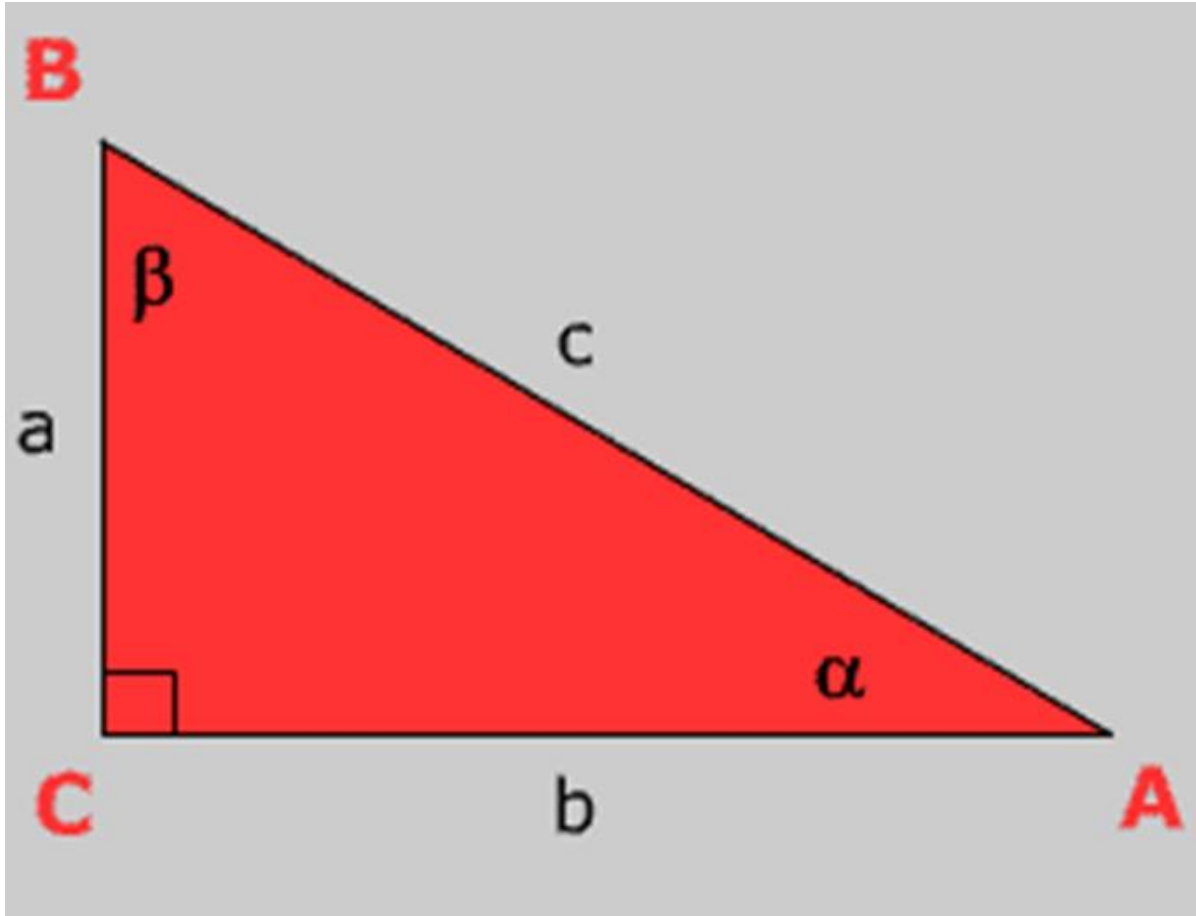
Pravokutni trokut



$$\cos \alpha = \frac{\text{priležea kateta}}{\text{hipotenuza}}$$

$$\cos \alpha = \frac{b}{c} \qquad \cos \beta = \frac{a}{c}$$

Pravokutni trokut



$$\operatorname{tg} \alpha = \frac{\text{nasuprotna kateta}}{\text{priležea kateta}}$$

$$\operatorname{tg} \alpha = \frac{a}{b} \qquad \operatorname{tg} \beta = \frac{b}{a}$$

Pravokutni trokut

Zadan je pravokutni trokut s katetom duljine 4 cm i kutom nasuprot kateti s mjerom 30° . Odredite duljine preostalih stranica i kutove.

$$\operatorname{tg} 30^\circ = \frac{4}{a}$$

$$a = \frac{4}{\operatorname{tg} 30^\circ}$$

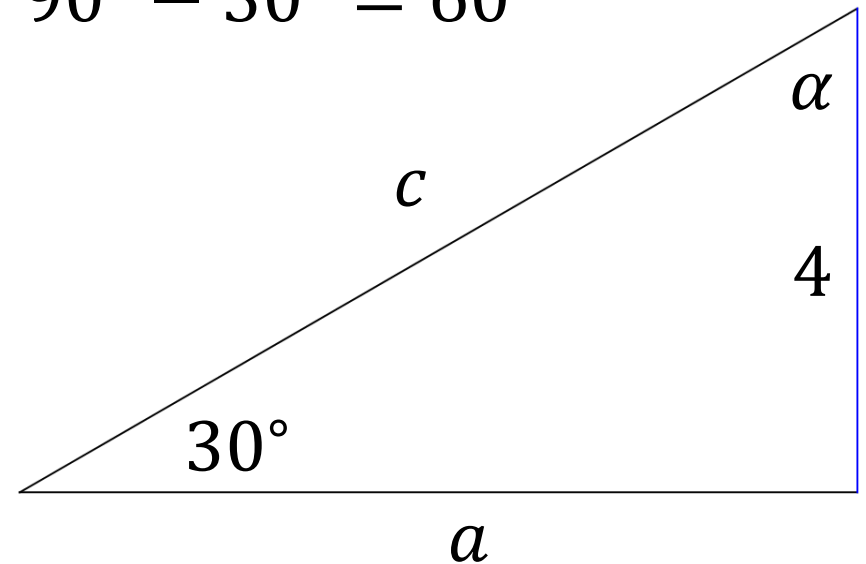
$$a = \frac{4}{\frac{1}{\sqrt{3}}} = 4\sqrt{3}$$

$$\sin 30^\circ = \frac{4}{c}$$

$$c = \frac{4}{\sin 30^\circ}$$

$$c = \frac{4}{\frac{1}{2}} = 8$$

$$\alpha = 90^\circ - 30^\circ = 60^\circ$$



Pravokutni trokut

U pravokutniku je poznata duljina dijagonale, koja iznosi 5 cm, te kut koji dijagonala zatvara s duljom stranicom, 40° . Odredite duljine stranica tog pravokutnika.

$$\cos 40^\circ = \frac{a}{5}$$

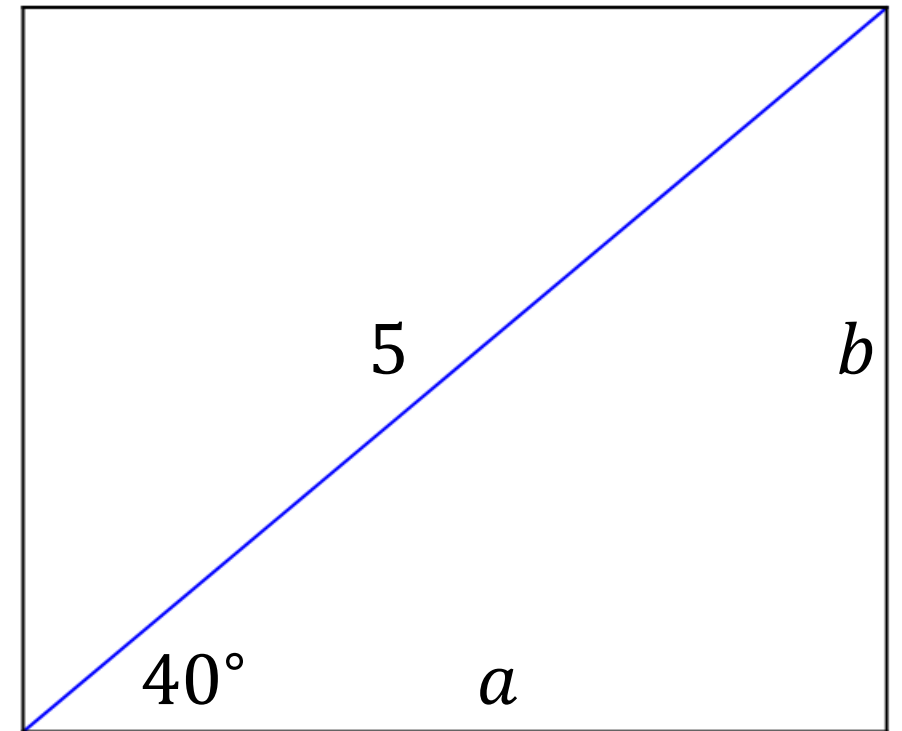
$$\sin 40^\circ = \frac{b}{5}$$

$$a = 5 \cos 40^\circ$$

$$b = 5 \sin 40^\circ$$

$$a = 5 \cdot 0.77 = 3.85$$

$$b = 5 \cdot 0.64 = 3.2$$



Hvala 😊