Ponavljanje za blic

$$A=\begin{bmatrix} 2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 1 \end{bmatrix} = 2\cdot(-1) = -2$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{array}\right] = 1+2-(4) = \underline{-1}$$

13.9

$y=x=f(x)$

$$\vec{r}=\begin{bmatrix} t \\ f(t) \end{bmatrix} \to \begin{bmatrix} t \\ t \end{bmatrix}$$

a) $\alpha=90^\circ$ b) $\alpha=60^\circ$ c) $\alpha=40^\circ$

c) $$\vec{r}'=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\cdot\vec{r}$$

$$\vec{r}'=\begin{bmatrix} \cos 40^\circ & -\sin 40^\circ \\ \sin 40^\circ & \cos 40^\circ \end{bmatrix}\cdot\begin{bmatrix} t \\ t \end{bmatrix}$$

$$\vec{r}'=\begin{bmatrix} 0.7660 & -0.6428 \\ 0.6428 & 0.7660 \end{bmatrix}\cdot\begin{bmatrix} t \\ t \end{bmatrix}$$

$$=\begin{bmatrix} 0.7660t-0.6428t \\ 0.6428t+0.7660t \end{bmatrix}$$

13.16

$\vec{r} = \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

a) $k_x = 2$

1. način

$\boxed{\begin{matrix} x \text{ se mijenja} \\ y \text{ isti} \end{matrix}}$

$\vec{r}' = \begin{bmatrix} 2+2t \\ 2-t \end{bmatrix}$

2. način

$\vec{r}' = S_x \cdot \vec{r}$

$= \begin{bmatrix} k_x & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 2+2t \\ 2-t \end{bmatrix}$

b) $k_y = \frac{1}{3}$

1. način x isti

y se mijenja

$= \begin{bmatrix} 1+t \\ \frac{2}{3} - \frac{1}{3}t \end{bmatrix}$

2. način

$\vec{r}'' = S_y \cdot \vec{r}$

$\vec{r}'' = \begin{bmatrix} 1 & 0 \\ 0 & k_y \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 1+t \\ \frac{2}{3} - \frac{1}{3}t \end{bmatrix}$

a)

$\vec{r} = \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

npr. $t = 0$, $t = 1$

$\begin{matrix} 1+0 = \\ 2-0 = \end{matrix} \boxed{\begin{matrix} 1 \\ 2 \end{matrix}}$ 1. točka $A(1, 2)$

$\begin{matrix} 1+1 = \\ 2-1 = \end{matrix} \boxed{\begin{matrix} 2 \\ 1 \end{matrix}}$ 2. točka $B(2, 1)$

$A(2, 2)$ $B(4, 1)$

13.11

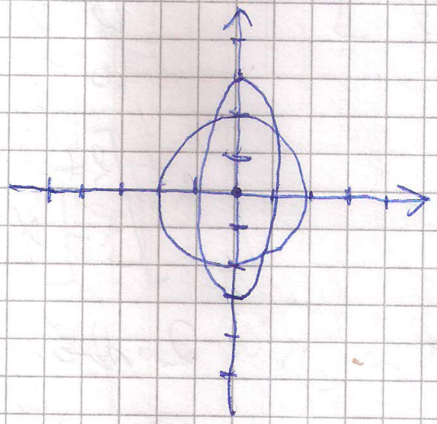
opěn: $\underline{(x-r)^2 + (y-q)^2 = R^2}$

$S(r, q)$

$S(0,0)$, $R = 2$

$$x^2 + y^2 = 4$$

vektorovo: $\vec{r} = \begin{bmatrix} R\cos t \\ R\sin t \end{bmatrix} \rightarrow \begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{1} + \frac{y^2}{9} = 1$$

$a = 1$ $\quad$ $b = \sqrt{9} = \underline{3}$

$\underline{k_x}$

$2 \rightarrow 1$

$2 \cdot k_x = 1 /:2$

$\boxed{k_x = \frac{1}{2}}$

$\underline{k_y}$

$2 \rightarrow 3$

$2 \cdot k_y = 3/2$

$k_y = \frac{3}{2} = \underline{1.5}$

14.1

a) $A(1, -2, 2)$ $\qquad$ $\vec{s} = \overrightarrow{AB}$ $\qquad$ $\underline{B \text{ mínus } A}$

$B(0, -3, 1)$

$\vec{r} = \vec{r}_A + t \cdot \vec{s}$

$\vec{r} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + t \cdot \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

$\vec{r} = \begin{bmatrix} 1 - t \\ -2 - t \\ 2 - t \end{bmatrix}$

b)

$$\frac{x-2}{3} = \frac{y+1}{0} = \frac{z-0}{-1} \rightarrow \text{smjer}$$

$$\frac{x-x_A}{s_x} = \frac{y-y_A}{s_y} = \frac{z-z_A}{s_z}$$

$$\vec{s} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix},\ A(2,-1,0) \rightarrow \text{ostali podatke}$$

$$\vec{r} = \vec{r}_A + \lambda \cdot \vec{s}$$

$$\vec{r} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \lambda \cdot \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 2+3\lambda \\ -1 \\ -\lambda \end{bmatrix},\ \lambda \in \mathbb{R}$$

c)

$A(3,-2,1)$

$$\vec{r} = \begin{bmatrix} 2-\lambda \\ 1+3\lambda \\ -2 \end{bmatrix}$$

posljednji pravci moraju biti jednaki ili proporcionalni vektoru smjera $\vec{s}$.

– neka su jednaki:

$$\vec{s} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\vec{r}' = \vec{r}_A + \lambda \cdot \vec{s}$$

$$\vec{r}' = \begin{vmatrix} 3 \\ -2 \\ 1 \end{vmatrix} + \lambda \cdot \begin{vmatrix} -1 \\ 3 \\ 0 \end{vmatrix}$$

$$\vec{r}' = \begin{bmatrix} 3-\lambda \\ -2+3\lambda \\ 1 \end{bmatrix}$$

14.2

a)

A(3, −1, 0)
B(−1, −2, 2)

$$\frac{x-3}{-4} = \frac{y+1}{-1} = \frac{z}{2}$$

$\vec{s} = \vec{AB}$

$\vec{s} = \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}$

b)

$\vec{r} = \begin{bmatrix} -t \\ 2 \\ 1-3t \end{bmatrix}$

A(0, 2, 1)

1. način

$\vec{s} = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$

... opet ko gore

14.3

a)

$\vec{r}_1 = \begin{bmatrix} 2-3t \\ 2+t \\ 3 \end{bmatrix}$

$\vec{r}_2 = \begin{bmatrix} 4-4t \\ 4 \\ -1+2t \end{bmatrix}$

izjednačimo

$$\begin{cases} 2-3t_1 = 4-4t_2 \\ 2+t_1 = 4 \\ 3 = -1+2t_2 \end{cases}$$

$2+t_1 = 4$

$t_1 = 2$

$3 = -1+2t_2$

$t_2 = 2$

t_1 t_2

$2-3\cdot 2 = 4-4\cdot 2$

$\boxed{-4 = -4}$ ✓

presek
to je
u(-4, 4

uvrstimo
ili t_2 u
ili t_1 u

$2-3\cdot 2 = -$

b)

$$\frac{x}{3} = \frac{y-5}{-2} = \frac{z-2}{1} \qquad A_1(0,5,2) \quad \vec{s_1} = (3,-2,1)$$

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+1}{1} \qquad A_2(1,2,-1) \quad \vec{s_2} = (2,1,1)$$

$$\vec{r_1} = \begin{bmatrix} 3t \\ 5-2t \\ 2+1t \end{bmatrix} \quad i \quad \vec{r_2} \begin{bmatrix} 1+2t \\ 2+t \\ -1+t \end{bmatrix}$$

$$\begin{cases} 3t_1 = 1+2t_2 \\ 5-2t_1 = 2+t_2 \\ 2+t_1 = -1+t_2 \end{cases}$$

$$2+t_1 = -1+t_2$$

$$t_1 = -3+t_2$$

$$3t_1 = 1+2t_2$$

$$3\cdot(-3+t_2) = 1+2t_2$$

$$-9+3t_2 = 1+2t_2$$

$$t_2 = 10$$

$$t_1 = -3+10$$

$$t_1 = 7$$

provjera

$$5-2t_1 = 2+t_2$$

$$5-2\cdot 7 = 2+10$$

$$-9 = 12$$

ne sijeku se

paralelni

$$\vec{s_1} \neq \lambda \cdot \vec{s_2}$$

mimoilazni su

14.4

a) $\vec{r_1} \begin{bmatrix} 2-t \\ 2+t \\ 3 \end{bmatrix} \quad \vec{r_2} = \begin{bmatrix} 4-\alpha t \\ 2t \\ -1+3\alpha t \end{bmatrix}$

seciraju okomito: $\vec{s_1}\cdot\vec{s_2} = 0$

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -\alpha \\ 2 \\ 3\alpha \end{bmatrix} = 0$$

$$-1\cdot(-\alpha)+1\cdot 2+(0\cdot 3\alpha) = 0$$

$$\alpha+2 = 0$$

$$\alpha = -2$$