

DIGITALNA LOGIKA

Booleova algebra

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- Neutralni element

a) $A + 0 = A$

b) $A * 1 = A$

Komplement

a) $A + \bar{A} = 1$

b) $A * \bar{A} = 0$

- Komutativnost

a) $A + B = B + A$

b) $A * B = B * A$

Distributivnost

a) $A * (B + C) = A * B + A * C$

b) $A + B * C = (A + B) * (A + C)$

Komutativnost	$A + B = B + A$	$A \cdot B = B \cdot A$
Asocijativnost	$(A + B) + C = A + (B + C)$	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
Distributivnost	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + BC = (A + B)(A + C)$
Neutralni element	$A + 0 = A$	$A \cdot 1 = A$
	$A + A = A$	$A \cdot A = A$
Komplementarnost	$A + \bar{A} = 1$	$A \cdot \bar{A} = 0$
De Morganovi zakoni	$\overline{A + B} = \bar{A} \cdot \bar{B}$	$\overline{A \cdot B} = \bar{A} + \bar{B}$
Involutivnost	$\bar{\bar{A}} = A$	
Anihilacija	$A + 1 = 1$	$A \cdot 0 = 0$
Apsorpcija	$A \cdot (A + B) = A$	$A + A \cdot B = A$

De Morganovi teoremi

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

A	B	A+B	$\overline{A+B}$	\bar{A}	\bar{B}	$\bar{A} \cdot \bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

A	B	A·B	$\overline{A \cdot B}$	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$$\begin{aligned} & \overline{A \cdot B} \cdot \bar{B} \cdot (\overline{1 \cdot B}) \\ &= (\bar{A} + \bar{B}) \cdot \bar{B} \cdot \bar{B} \\ &= (\bar{A} + \bar{B}) \cdot \bar{B} \\ &= \bar{A}\bar{B} + \bar{B} \\ &= \bar{B}(\bar{A} + 1) \\ &= \bar{B} \end{aligned}$$

primjena de Morganovih zakona

primjena pravila $\bar{B} \cdot \bar{B} = \bar{B}$

primjena pravila $\bar{B} \cdot \bar{B} = \bar{B}$

primjena pravila $(\bar{A} + 1) = 1$

1. Dokazati da vrijedi

$$A * (\bar{A} + A * B) = A * B$$

- $A * (\bar{A} + A * B) =$
- $= A * ((\bar{A} + A) * (\bar{A} + B))$ -drugi zakon distribucije
- $= A * (1 * (\bar{A} + B))$ -komplement
- $= A * ((\bar{A} + B) * 1)$ -komutacija
- $= A * (\bar{A} + B)$ -neutralni element
- $= (A * \bar{A}) + (A * B)$ -prvi zakon distribucije
- $= 0 + (A * B)$ -komplement
- $= (A * B) + 0$ -komutacija
- $= A * B$ -neutralni element

2. Pomoću pravila logičke algebre minimizirati logičku funkciju:

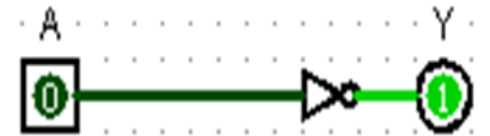
$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$

$$Y = \bar{A}\bar{B}(\bar{C} + C) + \bar{A}B(\bar{C} + C)$$

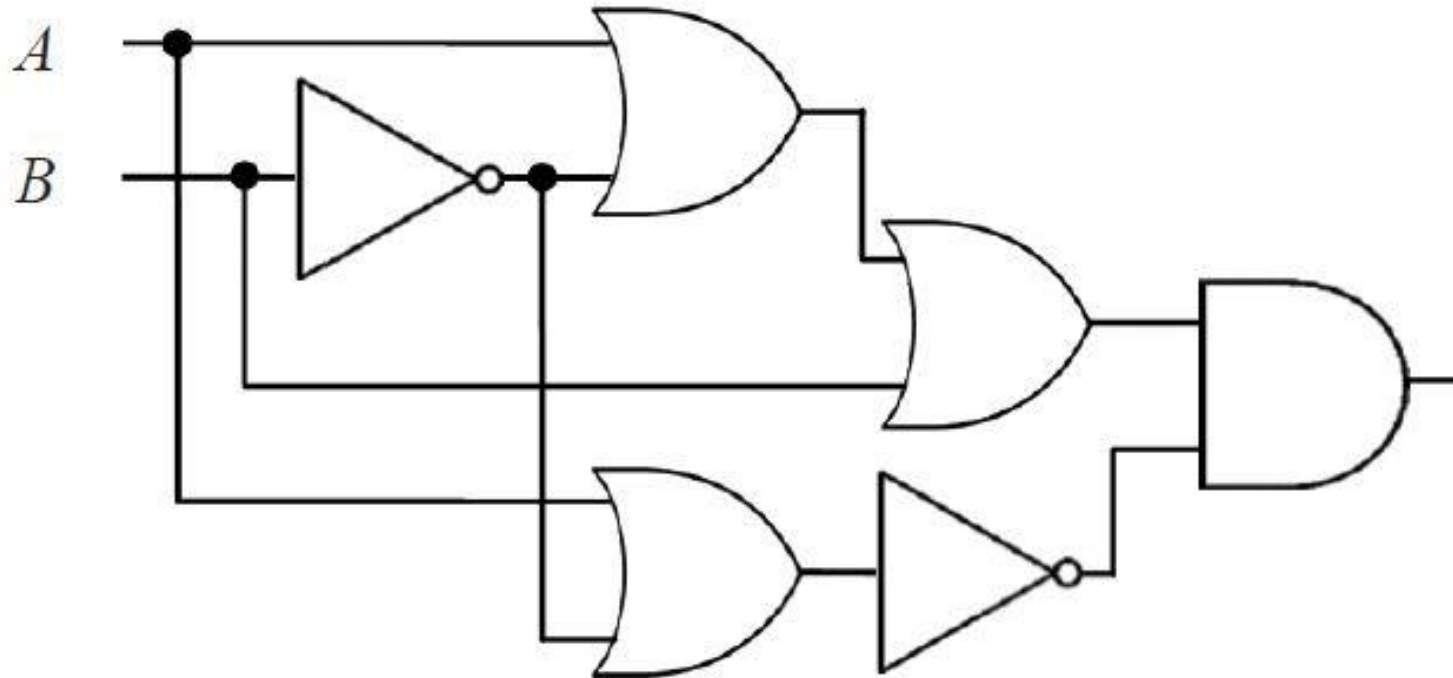
$$Y = \bar{A}\bar{B} * 1 + \bar{A}B * 1$$

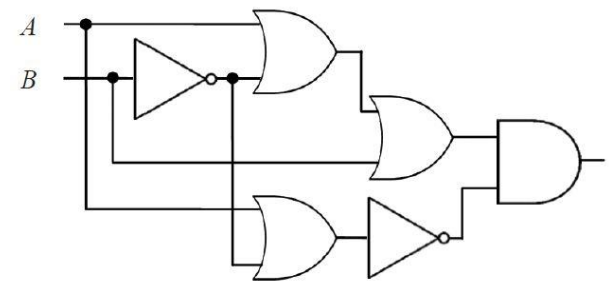
$$Y = \bar{A}(\bar{B} + B)$$

$$Y = \bar{A} * 1 = \bar{A}$$



3. Napisati izraz za ovu logičku shemu. Potom izraz minimizirati, te nacrtati pojednostavljenu shemu.

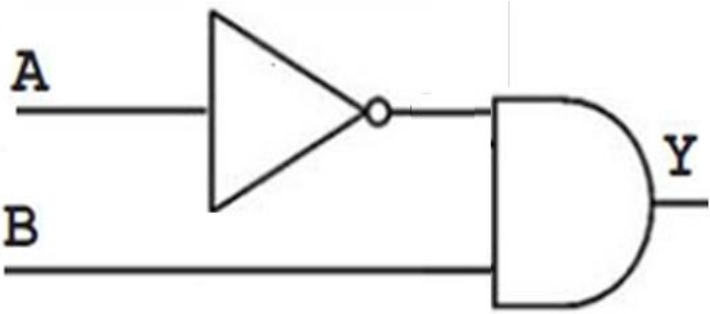




$$(A + \bar{B} + B) \cdot (\overline{\overline{A + \bar{B}}})$$

$$1 \cdot (\bar{A} \cdot \bar{\bar{B}})$$

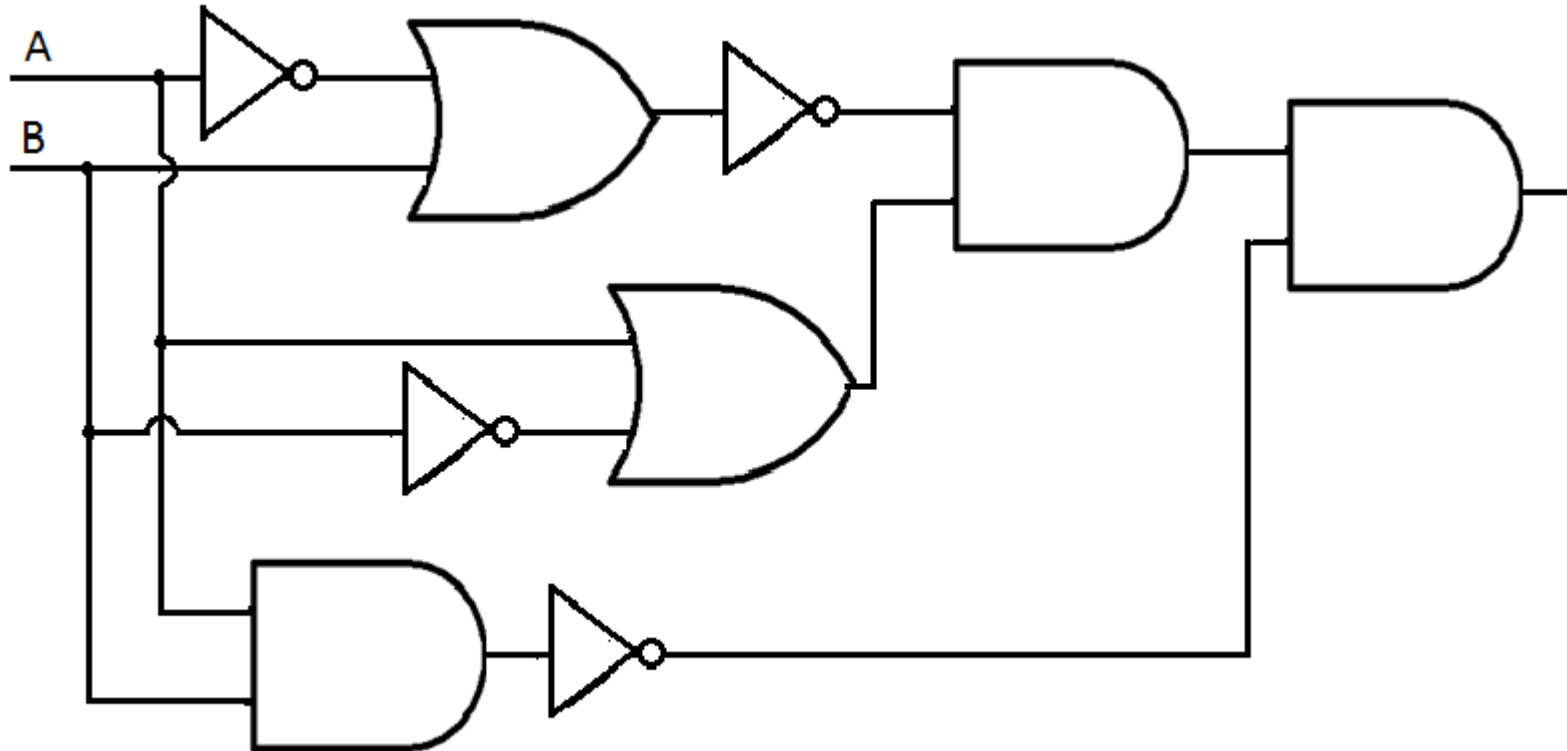
$$\bar{A} \cdot B$$



4. Na osnovu zadanog logičkog izraza **nacrtati** pripadajući **logičku shemu**. Potom minimizirati logički izraz. Nacrtati logičku shemu minimiziranog izraza.

$$\overline{A} \cdot B \cdot (A + \overline{B}) \cdot (\overline{\overline{A} + B})$$

$$\overline{A} \cdot B \cdot (A + \overline{B}) \cdot \overline{\overline{A} + B}$$



$$\overline{A} \cdot B \cdot (A + \overline{B}) \cdot \overline{(\overline{A} + B)}$$

$$(\overline{A} + \overline{B}) \cdot (A + \overline{B}) \cdot (\overline{\overline{A}} \cdot \overline{B})$$

$$(\overline{A} \cdot A + \overline{A} \cdot \overline{B} + \overline{B} \cdot A + \overline{B} \cdot \overline{B}) \cdot (A \cdot \overline{B})$$

$$(\overline{A} \cdot \overline{B} + \overline{B} \cdot A + \overline{B}) \cdot (A \cdot \overline{B})$$

$$\overline{B} \cdot (\overline{A} + A + 1) \cdot (A \cdot \overline{B})$$

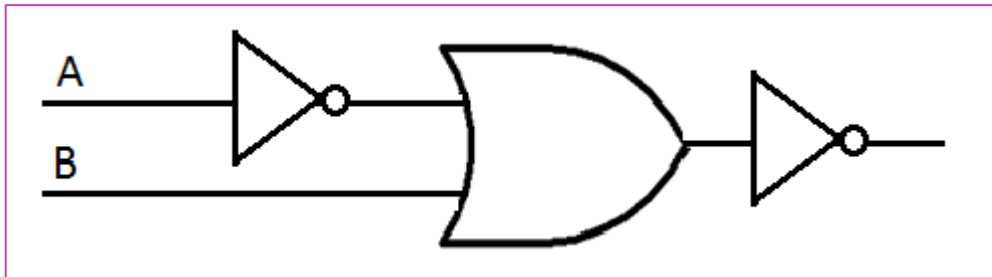
$$\overline{B} \cdot A \cdot \overline{B}$$

$$A \cdot \overline{B}$$

$$A \cdot \bar{B}$$

$$\overline{\overline{A \cdot \bar{B}}}$$

$$\overline{\overline{A}} + \overline{\overline{\bar{B}}} = \bar{A} + B$$



5. Zadani logički izraz minimizirati, a potom nacrtati minimiziranu logičku shemu.

$$(A \cdot B + \overline{C}) \cdot (\overline{A \cdot \overline{B}} + C) + (A \cdot \overline{C})$$

$$(A \cdot B + \overline{C}) \cdot (\overline{A \cdot B} + C) + (A \cdot \overline{C})$$

$$(A \cdot B + \overline{C}) \cdot (\overline{A} + B + C) + (A \cdot \overline{C})$$

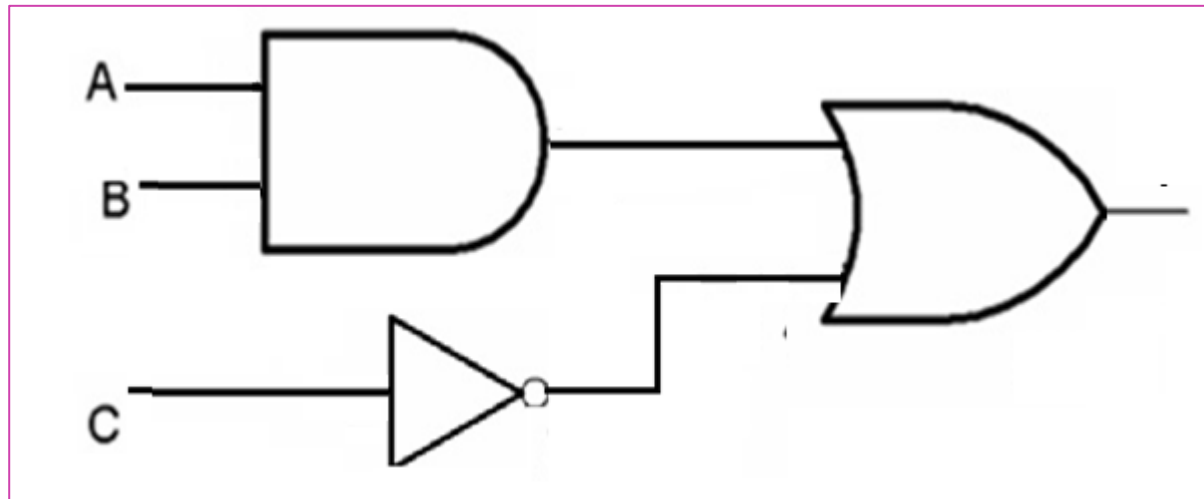
$$(A \cdot B \cdot \overline{A} + A \cdot B \cdot B + A \cdot B \cdot C + \overline{C} \cdot \overline{A} + \overline{C} \cdot B + \overline{C} \cdot C) + A \cdot \overline{C}$$

$$(A \cdot B + A \cdot B \cdot C + \overline{C} \cdot \overline{A} + \overline{C} \cdot B) + A \cdot \overline{C}$$

$$A \cdot B \cdot (1 + C) + \overline{C} \cdot (\overline{A} + B + A)$$

$$A \cdot B + \overline{C}$$

$$A \cdot B + \bar{C}$$

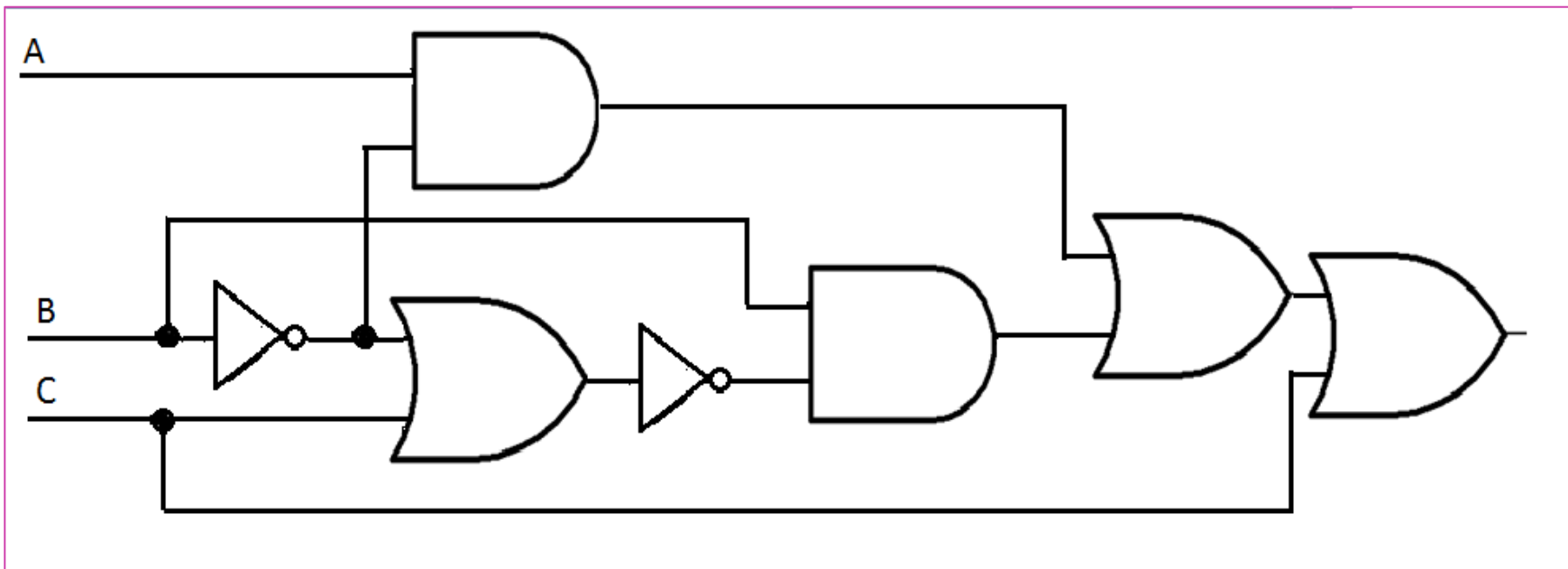


6. Na osnovu zadanog logičkog izraza **nacrtati** pripadajuću **logičku shemu**.

- Zadani logički izraz minimizirati, te **nacrtati** minimiziranu **logičku shemu**.

$$\overline{\overline{B + C} \cdot B + A \cdot \overline{B + C}}$$

$$\overline{\overline{B + C} \cdot B + A \cdot \overline{B} + C}$$



$$\overline{\overline{B + C} \cdot B + A \cdot \overline{B} + C}$$

$$B \cdot \overline{C} \cdot B + A \cdot \overline{B} + C$$

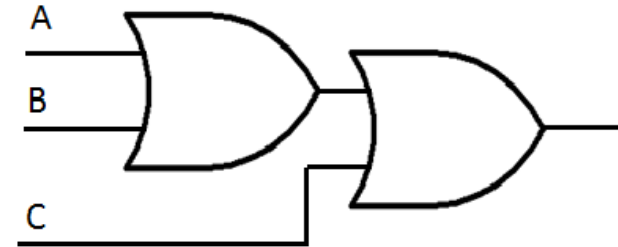
$$B \cdot \overline{C} + C + A \cdot \overline{B}$$

$$(B + C) \cdot (\overline{C} + C) + A \cdot \overline{B}$$

$$B + C + A \cdot \overline{B}$$

$$(B + A) \cdot (B + \overline{B}) + C$$

$$B + A + C$$



7. Pomoću pravila logičke algebre minimizirati logičku funkciju:

$$\begin{aligned} Y &= \overline{A(\overline{B} + C) + B(AC + B)} \\ &= A(\overline{B} + C) \cdot \overline{B(AC + B)} = \\ &= A(\overline{B} + C) \cdot \overline{ABC + BB} = \\ &= A(\overline{B} + C) \cdot \overline{ABC + B} = \\ &= A(\overline{B} + C) \cdot \overline{B(AC + 1)} = \\ &= \overline{A} \overline{B} (\overline{B} + C) = \overline{A} \overline{B} \overline{B} + \overline{A} \overline{B} C = \\ &= \overline{A} \overline{B} + \overline{A} \overline{B} C = \overline{A} \overline{B} (1 + C) = \overline{A} \overline{B} \end{aligned}$$

8. Pomoću pravila logičke algebre minimizirati logičku funkciju:

$$\begin{aligned}
 Y &= (A + D) \cdot \overline{A} \overline{B} \overline{C} + \overline{C + D} \cdot \overline{B} + \overline{A D} \\
 &= (A + D) \overline{A} \overline{B} \overline{C} + \overline{C} \cdot \overline{D} \cdot \overline{\overline{B}} \overline{A} \overline{D} && \text{Dvostruki komplement} \\
 &= (A + D) \overline{A} \overline{B} \overline{C} + \overline{C} \overline{D} \overline{B} (\overline{A + D}) && \text{De Morganovo pravilo} \\
 &= \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} \overline{D} && \text{De Morganovo pravilo} \\
 &= \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} \overline{D} && = 0 \\
 &= \overline{A} \overline{B} \overline{C} (D + \overline{D}) + \overline{A} \overline{B} \overline{C} \overline{D} && = 1 \\
 &= \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} \overline{D}
 \end{aligned}$$

9. Pomoću pravila logičke algebre minimizirati logičku funkciju:

$$Y = \overline{A + \overline{B}} \cdot (\overline{A} \overline{B} + C) \cdot (B + \overline{C})$$

$$= \overline{A + \overline{B}} \cdot (\overline{A} \overline{B} \overline{B} + BC + \overline{A} \overline{B} \overline{C} + C \overline{C}) =$$

$$\overline{A} \overline{B} (BC + \overline{A} \overline{B} \overline{C}) =$$

$$\overline{A} \overline{B} BC + \overline{A} \overline{A} \overline{B} \overline{C} = \overline{A} \overline{B} C$$

Zadaci za vježbu:

Napisati drugu stranu slijedećih zakona (aksioma i teorema)

Booleove algebre:

$$A + \bar{A} =$$

$$A \overline{B+C} =$$

$$A + 0 =$$

$$A + \bar{A}B =$$

$$\overline{A * B * C} =$$

$$A * \bar{A} =$$

Pomoću pravila Booleove algebre pojednostavniti zadane logičke funkcije:

$$Y = A + B \overline{(A + B * C)}$$

$$Y = \overline{A * B} + A * C + \bar{B}$$

$$Y = \bar{A} + B(\overline{AB} + C) + \bar{A}B$$