1. **[I1, 2 boda]** Pomnožite i podijelite polinome $f(x)$ *i* $g(x)$:

$$f(x) = 2x^3 - 3x + 2 \qquad g(x) = -x + 2$$

$$\begin{aligned} f(x)\cdot g(x) &= (2x^3 - 3x + 2)(-x + 2) \\ &= -2x^4 + 4x^3 + 3x^2 - 6x - 2x + 4 \\ &= -2x^4 + 4x^3 + 3x^2 - 8x + 4 \end{aligned}$$

$f(x) : g(x) = ?$

$$(2x^3 \,\_\_\, - 3x + 2) : (-x + 2) = -2x^2 - 4x - 5 + \frac{12}{-x+2}$$

$$\begin{array}{l} \underline{-2x^3 \pm 4x^2} \\ \quad 4x^2 - 3x + 2 \\ \quad \underline{-4x^2 \pm 8x} \\ \qquad\quad 5x + 2 \\ \qquad\quad \underline{-5x \pm 10} \\ \qquad\qquad 12 \end{array}$$

12 ← ostatak

$= \text{rezultat} + \dfrac{\text{ostatak}}{\text{djelitelj}}$

2. **[I1, 2 boda]** Skicirajte krivulje:

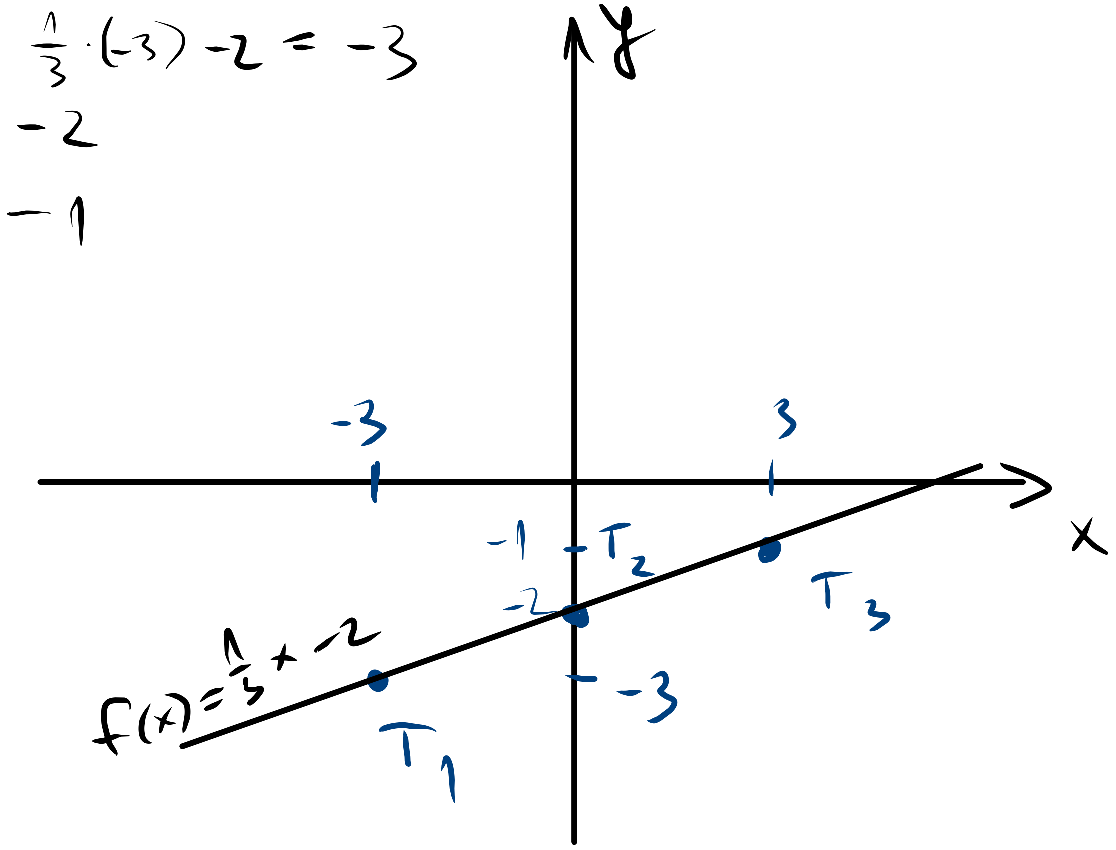
a) $f(x) = \frac{1}{3}x - 2$ ← PRAVAC

b) $f(x) = -2x^2 + 7x - 3$

↖ PARABOLA

a) $f(x) = \frac{1}{3}x - 2$

	x	$f(x)$
$T_1(-3,-3)$	-3	$\frac{1}{3}\cdot(-3)-2=-3$
$T_2(0,-2)$	0	-2
$T_3(3,-1)$	3	-1



b) $f(x) = -2x^2 + 7x - 3$

$a = -2$ ← $a<0$ ∩ ~~$a>0$ ∪~~

$b = 7$

$c = -3$ → odsječak na y-osi

Nultočke

$$x_{1,2} = \frac{-7 \pm \sqrt{49-24}}{-4}$$

$$x_{1,2} = \frac{-7 \pm 5}{-4}$$

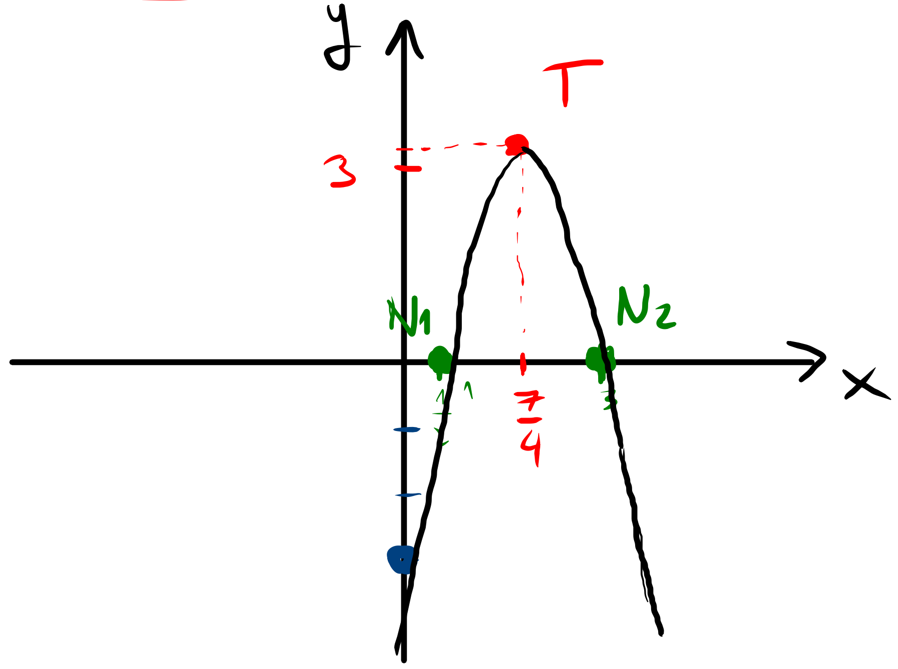
$\boxed{x_1 = \frac{1}{2}}$ $\boxed{x_2 = 3}$

$N_1\left(\frac{1}{2}, 0\right)$ $N_2(3, 0)$

Tjeme $T\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

$$T\left(-\frac{7}{-4}, \frac{24-49}{-8}\right)$$

$$\boxed{T\left(\frac{7}{4}, \frac{25}{8}\right)}$$



3. Nađite rješenja nejednadžbi:

a) **[I1, 1 bod]** $(2x-1)^2 \geq 2x(2x+3)$

b) **[I1, 2 boda]** $6x^2 + 3x < 2x + 1$

c) **[I1, 2 boda]** $\frac{2}{x+1} - \frac{3}{x-1} \leq 0$

Algebarski izrazi:

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

a) $(2x-1)^2 \geq 2x(2x+3)$

$\cancel{4x^2} - 4x + 1 \geq \cancel{4x^2} + 6x$

$-10x \geq -1 \quad / : (-10)$

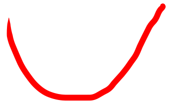
$\boxed{x \leq \frac{1}{10}}$

oprez kod dijeljenja i množenja nejednadžbe s negativnim brojem znak se OKREĆE

b) $6x^2 + 3x < 2x + 1$

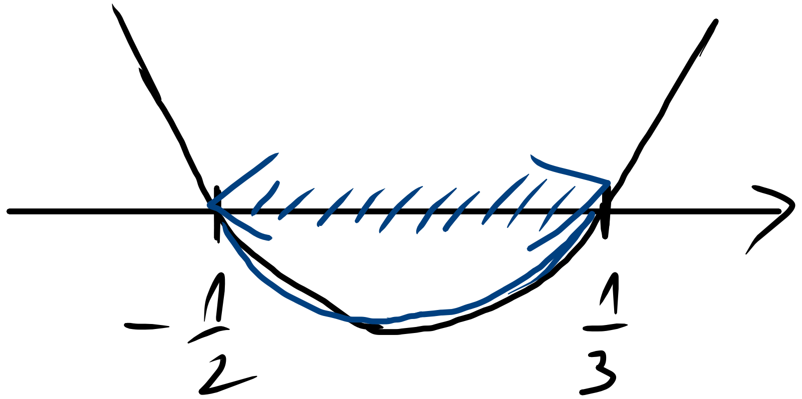
$6x^2 + x - 1 < 0$

$a = 6 > 0$



$x_{1,2} = \frac{-1 \pm \sqrt{1+24}}{12}$

$\boxed{x_1 = \frac{1}{3}}$ $\boxed{x_2 = -\frac{1}{2}}$



$\boxed{x \in \langle -\frac{1}{2}, \frac{1}{3} \rangle}$

3. Nađite rješenja nejednadžbi:

a) **[I1, 1 bod]** $(2x-1)^2 \geq 2x(2x+3)$

b) **[I1, 2 boda]** $6x^2 + 3x < 2x + 1$

c) **[I1, 2 boda]** $\frac{2}{x+1} - \frac{2}{x-1} \leq 0$

c) $\frac{2}{x+1} - \frac{2}{x-1} \leq 0 \quad / \cdot (x+1)^2 \cdot (x-1)^2$

Uvjeti:

$x+1 \neq 0$ $\quad$ $x-1 \neq 0$

$\boxed{x \neq -1}$ $\quad$ $\boxed{x \neq 1}$

$\frac{2}{\cancel{x+1}}(x+1)^{\cancel{2}}(x-1)^2 - \frac{2}{\cancel{x-1}}(x+1)^2(x-1)^{\cancel{2}} \leq 0$

$2(x+1)(x-1)^2 - 2(x+1)^2(x-1) \leq 0$

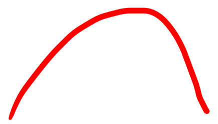
$(2x+2)(x^2-2x+1) - (x^2+2x+1)(2x-2) \leq 0$

$2x^3 - 4x^2 + 2x + 2x^2 - 4x + 2 - (2x^3 - 2x^2 + 4x^2 - 4x + 2x - 2) \leq 0$

$\cancel{2x^3} - 2x^2 \cancel{- 2x} + 2 \cancel{- 2x^3} - 2x^2 \cancel{+ 2x} + 2 \leq 0$

$-4x^2 + 4 \leq 0$

$a = -4 < 0$



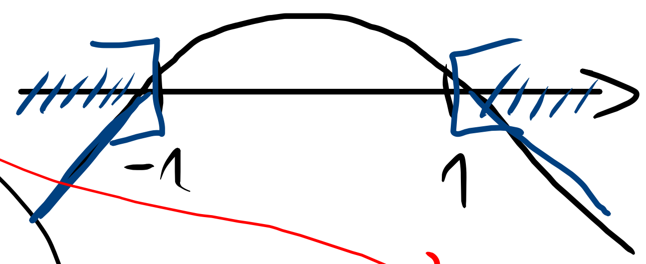
Nultočke

$x_{1,2} = \frac{0 \pm \sqrt{0+64}}{-8}$

$x_{1,2} = \frac{\pm 8}{-8}$

$\boxed{x_1 = -1}$

$\boxed{x_2 = 1}$



$\boxed{x \in \langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle}$

3. Nađite rješenja nejednadžbi:

a) **[I1, 1 bod]** $(2x-1)^2 \geq 2x(2x+3)$

b) **[I1, 2 boda]** $6x^2 + 3x < 2x + 1$

c) **[I1, 2 boda]** $\frac{2}{x+1} - \frac{2}{x-1} \leq 0$

c) 2. NAČIN

$$\frac{2}{x+1} - \frac{2}{x-1} \leq 0$$

$$\frac{2(x-1) - 2(x+1)}{(x+1)(x-1)} \leq 0$$

$$\frac{2x - 2 - 2x - 2}{(x+1)(x-1)} \leq 0$$

$$\frac{-4}{(x+1)(x-1)} \leq 0$$

$x = -1$ $\quad$ $x = 1$

	$-\infty$	-1	1	$+\infty$
-4	$-$		$-$	$-$
$x+1$	$-$	0	$+$	$+$
$x-1$	$-$		$-$ 0	$+$
$(x+1)(x-1)$	$+$		$-$	$+$
$\frac{-4}{(x+1)(x-1)}$	$-$	0	$+$ 0	$-$

$$\boxed{x \in \langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle}$$

4. **[I1, 4 boda]** Odredite domenu i nultočke sljedećih funkcija:

a) $f(x) = \sqrt{x-1} - \sqrt{4-2x}$

b) $f(x) = \frac{3^x - 9}{x^2 + x + 1}$

a) $f(x) = \sqrt{x-1} - \sqrt{4-2x}$

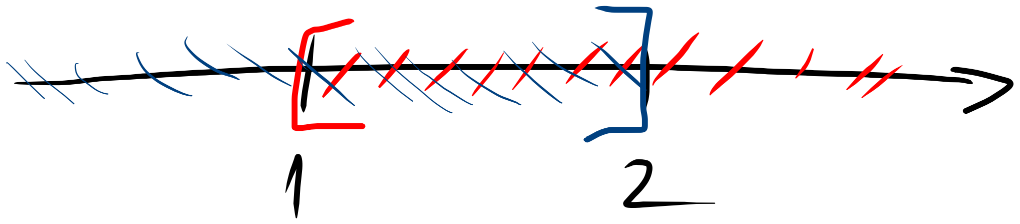
$x - 1 \geq 0$

$\boxed{x \geq 1}$

$4 - 2x \geq 0$

$-2x \geq -4 \quad / :(-2)$

$\boxed{x \leq 2}$



$\boxed{D_f = [1, 2]}$

Nultočke $f(x) = 0$

$0 = \sqrt{x-1} - \sqrt{4-2x}$

$\sqrt{4-2x} = \sqrt{x-1} \quad /^2$

$4 - 2x = x - 1$

$-3x = -5$

$\boxed{x = \frac{5}{3}}$ ✓ u D_f je

$\left[\frac{3}{3}, \frac{6}{3}\right]$

1 2

4. **[I1, 4 boda]** Odredite domenu i nultočke sljedećih funkcija:

a) $f(x) = \sqrt{x-1} - \sqrt{4-2x}$

b) $f(x) = \frac{3^x-9}{x^2+x+1}$

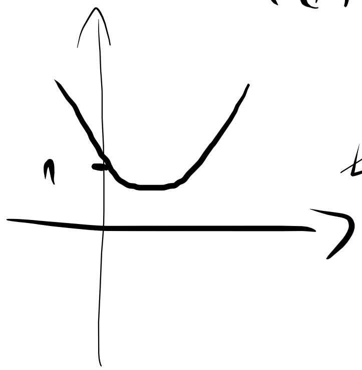
$$x^2+x+1 \neq 0$$

$$x_{1,2} \neq \frac{-1 \pm \sqrt{1-4}}{2}$$

$$x_{1,2} \neq \frac{-1 \pm \sqrt{-3}}{2}$$

nema realnih rješenja

$f(x) = x^2+x+1$ nikada neće biti 0



$$\boxed{D_f = \mathbb{R}}$$

Nultočke $f(x) = 0$

$$0 = \frac{3^x-9}{x^2+x+1}$$

RAZLOMAK JE 0 SAMO KADA JE BROJNIK 0.

$$3^x - 9 = 0$$

$$3^x = 9$$

$$3^x = 3^2$$

$$\boxed{x = 2}$$

✓ u domeni je

5. **[I1, 4 boda]** Riješite sljedeće jednadžbe:

a) $3^{x-1} \cdot 2^{x+1} = 1000$ (rješenje iskažite pomoću dekadskog logaritma)

b) $3 \ln \frac{2x-4}{3} = -6$

a) $3^{x-1} \cdot 2^{x+1} = 1000$

$3^x \cdot 3^{-1} \cdot 2^x \cdot 2^1 = 1000$

$a^n \cdot b^n = (a \cdot b)^n$

$\frac{1}{3} \cdot 2 \cdot 3^x \cdot 2^x = 1000$

$3^x \cdot 2^x = (3 \cdot 2)^x$

$\frac{2}{3} \cdot 6^x = 1000 \quad / \cdot \frac{3}{2}$

$\boxed{a^x = b \quad \Downarrow \quad x = \log_a b}$

$6^x = 1500$

$x = \log_6 1500$

$x = \log_6 1500$

$\log_a x = \frac{\log_c x}{\log_c a}$

$x = \frac{\log_{10} 1500}{\log_{10} 6}$

$\boxed{x = \frac{\log 1500}{\log 6}}$

b) $3\ln\frac{2x-4}{3} = -6$

$$3\cdot\ln\left(\frac{2x-4}{3}\right) = -6 \quad / :3$$

$$\ln\left(\frac{2x-4}{3}\right) = -2$$

$$\log_e\left(\frac{2x-4}{3}\right) = -2$$

$$\frac{2x-4}{3} = e^{-2} \quad / \cdot 3$$

$$2x-4 = 3e^{-2}$$

$$2x = 3e^{-2}+4 \quad / :2$$

$$\frac{2x-4}{3} > 0 \quad / \cdot 3$$

$$2x-4 > 0$$

$$2x > 4$$

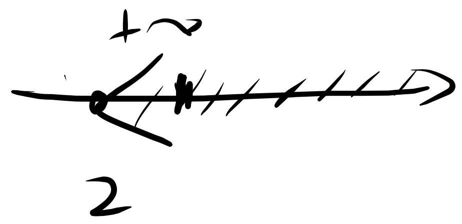
$$\boxed{x > 2}$$

$$\boxed{D_f = \langle 2, +\infty \rangle}$$

$$x = \frac{3e^{-2}}{2} + \frac{4}{2}$$

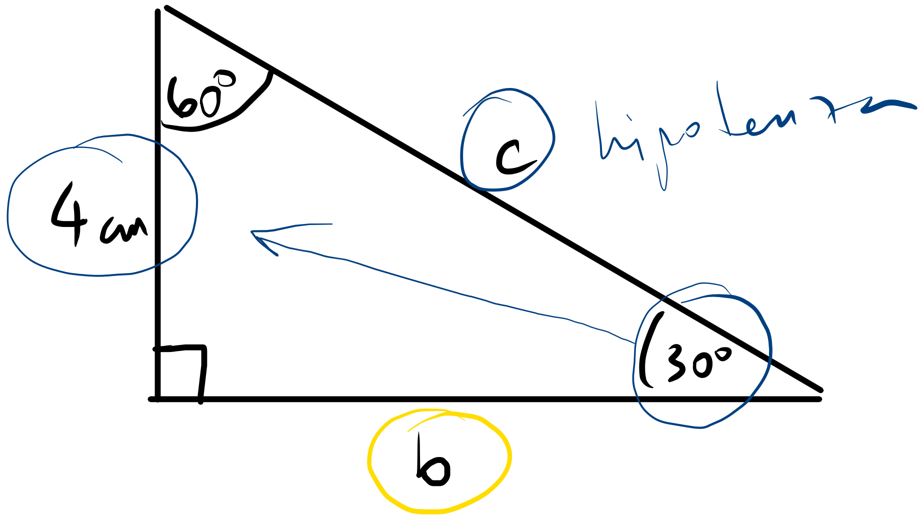
$$x = \underbrace{\frac{3e^{-2}}{2}}_{0} + 2$$

$$\boxed{x = \frac{3e^{-2}+4}{2}} \quad \checkmark$$



$\alpha = 30^\circ \quad \beta = 60^\circ \quad \gamma = 90^\circ$

6. **[I1, 3 boda]** Omjer veličina unutarnjih kutova je ~~1~~ : ~~2~~ : ~~3~~, a najkraća stranica je duljine 4 cm. Odredite veličine unutarnjih kutova i duljine preostalih stranica.



$$\sin\alpha = \frac{\text{nas.}}{\text{hip.}}$$

$$\sin 30^\circ = \frac{4}{c} \quad / \cdot c$$

$$c \sin 30^\circ = 4 \quad / : \sin 30^\circ$$

$$c = \frac{4}{\sin 30^\circ}$$

$$\boxed{c = \frac{4}{0{,}5} = 8 \text{ cm}}$$

$$c^2 = a^2 + b^2$$

$$8^2 = 4^2 + b^2$$

$$b^2 = 8^2 - 4^2$$

$$\boxed{b = \sqrt{64 - 16} = \sqrt{48} \text{ cm}}$$

Ishod učenja 2 – 20 bodova

1. **[I2, 2 boda]** Zadane su funkcije $f(x) = \ln(2x)$ i $g(x) = 2^{3x}$. Odredite $(f \circ g)(x)$ i $(g \circ f)(x)$

$$f \circ g(x) = f(g(x)) = \ln(2 \cdot 2^{3x})$$

$$g \circ f(x) = g(f(x)) = 2^{3\ln(2x)}$$

2. **[I2, 3 boda]** Pronađite inverz zadane funkcije.

$$f(x) = \sqrt[3]{x^2 - 1}\,.$$

Provjerite inverz kompozicijom.

$$y = \sqrt[3]{x^2 - 1} \quad /^3$$

$$y^3 = x^2 - 1$$

$$x^2 = y^3 + 1 \quad /\sqrt{}$$

$$x = \pm\sqrt{y^3 + 1}$$

$$\boxed{f^{-1}(x) = \sqrt{x^3 + 1}}$$

$$\left(\begin{aligned} x^2 &= 4 \quad /\sqrt{} \\ x_{1,2} &= \pm 2 \end{aligned} \right)$$

$$f^{-1} \circ f(x) = x$$

$$f \circ f^{-1}(x) = x$$

dovoljno samo jedno

$$f \circ f^{-1}(x) = f(f^{-1}(x)) = \sqrt[3]{\left(\sqrt{x^3+1}\right)^2 - 1} = \sqrt[3]{x^3 + 1 - 1} = \sqrt[3]{x^3} = x \checkmark$$

3. **[I2, 3 boda]** Za funkcije $f(x) = \frac{2}{x} + 1$ i $g(x) = x^2 + 1$ pronađite rješenja jednadžbe $(f \circ g)(x) = 2$.

$$f \circ g(x) = 2$$

$$f(g(x)) = 2$$

$$\frac{2}{x^2+1} + 1 = 2$$

$$x^2 + 1 \neq 0$$

$$x^2 \neq -1$$

$$\boxed{D_f = \mathbb{R}}$$

$$\frac{2}{x^2+1} = 1 \quad / \cdot (x^2+1)$$

$$2 = x^2 + 1$$

$$x^2 = 1 \quad / \sqrt{\ }$$

$$\boxed{x = \pm 1}$$

✓

4. **[I2, 3 boda]** Zadani su skupovi $A = \{1, \{2\}\}$ i $B = \{1, \{2, 3\}\}$

a) Koliko iznosi kardinalni broj $|B|$?

b) Zapišite partitivni skup $\mathcal{P}(A)$.

c) Zapišite Kartezijev produkt $B \times A$.

a) $|B| = \text{card}(B) = \begin{pmatrix}\text{broj elemenata} \\ \text{skupa } B\end{pmatrix}$

$|B| = 2$

$A = \{1, \{2\}\} \to$ elementi su $1, \{2\}$

b) $\mathcal{P}(A) = \{\emptyset, \{1\}, \{\{2\}\}, \{1, \{2\}\}\}$

c) $B \times A = \{(1, 1), (1, \{2\}), (\{2,3\}, 1), (\{2,3\}, \{2\})\}$

elementi B su 1 i $\{2,3\}$

elementi A su 1 i $\{2\}$

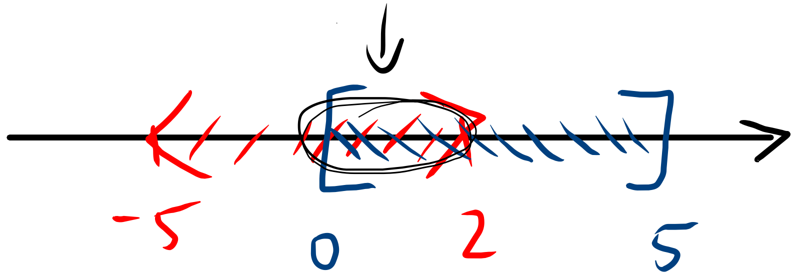
5. **[I2, 2 boda]** Zadani skupovi $A = \langle -5, 2\rangle$, $B = [0, 5]$, $C = \{-5, 0, 2, 5\}$. Odredite:

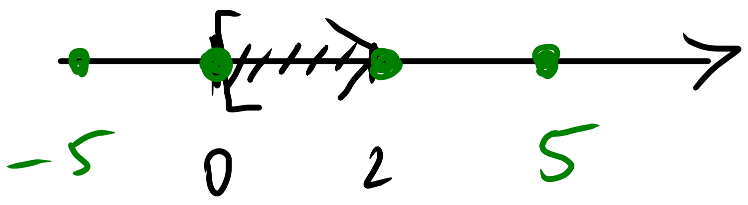
a) $(A \cap B) \backslash C$

b) $(A \cup C) \backslash B$

a)

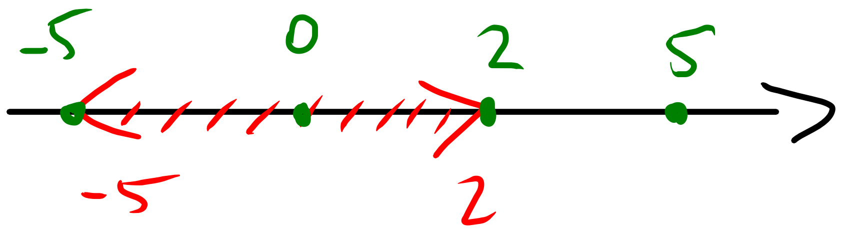
$$(A \cap B) \backslash C = [0, 2\rangle \backslash C = \langle 0, 2\rangle$$

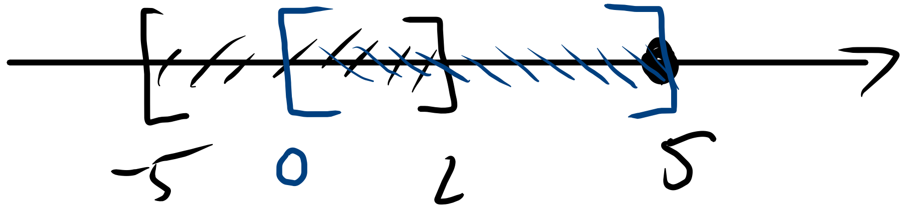




b)

$$(A \cup C) \backslash B = \left([-5, 2] \cup \{5\}\right) \backslash B = [-5, 0\rangle$$





6. **[I2, 1 bod]** Odredite 4. član niza ako je $a_1 = -2,\ a_2 = 1,\ a_n = 2a_{n-1} - n \cdot a_{n-2}$.

$a_1 = -2$

$a_2 = 1$

$a_n = 2a_{n-1} - n \cdot a_{n-2}$

$a_3 = 2 \cdot a_{3-1} - 3 \cdot a_{3-2} = 2 \cdot a_2 - 3 \cdot a_1$

$= 2 \cdot 1 - 3 \cdot (-2)$

$= 2 + 6 = 8$

$\underline{a_4} = 2 \cdot a_{4-1} - 4 \cdot a_{4-2} = 2 \cdot a_3 - 4 \cdot a_2$

$= 2 \cdot 8 - 4 \cdot 1$

$= 16 - 4 = \underline{12}$

7. **[I2, 3 boda]** Ako je za aritmetički niz zadano: $a_1 + a_3 + a_4 = 3$ i $a_3 - a_1 = 1$ odredite 11. član niza.

$a_{11} = ?$

$$\boxed{a_n = a_1 + (n-1)d}$$

$$a_1 + a_3 + a_4 = 3$$

$$a_3 - a_1 = 1$$

$$a_1 + a_1 + 2d + a_1 + 3d = 3$$

$$a_1 + 2d - a_1 = 1$$

$$3a_1 + 5d = 3$$

$$2d = 1$$

$$\boxed{d = \frac{1}{2}}$$

$$3a_1 + \frac{5}{2} = 3$$

$$3a_1 = 3 - \frac{5}{2}$$

$$3a_1 = \frac{1}{2} \quad \Big| :3$$

$$\boxed{a_1 = \frac{1}{6}}$$

$$a_{11} = a_1 + (11-1)d$$

$$= \frac{1}{6} + 10 \cdot \frac{1}{2}$$

$$= \frac{1}{6} + 5$$

$$\boxed{a_{11} = \frac{1+30}{6} = \frac{31}{6}}$$

8. **[I2, 3 boda]** Odredite zbroj prvih 5 članova geometrijskog niza za kojeg vrijedi $a_1, q > 0$, $a_1 \cdot a_3 = 9$ i $a_3 + 2a_4 = 3$.

$$a_1 \cdot a_3 = 9$$
$$a_3 + 2a_4 = 3$$

$$\boxed{a_n = a_1 \cdot q^{n-1}}$$

$$a_1 \cdot a_1 \cdot q^2 = 9$$
$$a_1 \cdot q^2 + 2 \cdot a_1 \cdot q^3 = 3$$

$$a_1^2 \cdot q^2 = 9 \quad / \sqrt{\ }$$
$$a_1 \cdot q = 3 \quad | : q$$
$$\boxed{a_1 = \frac{3}{q}} = \frac{3}{\frac{1}{2}} = \boxed{6}$$

$$\frac{3}{\cancel{q}} \cdot q^{\cancel{2}} + 2 \cdot \frac{3}{\cancel{q}} \cdot q^{\cancel{3}\,2} = 3$$
$$3q + 6q^2 = 3$$
$$6q^2 + 3q - 3 = 0$$
$$q_{1,2} = \frac{-3 \pm \sqrt{9 + 72}}{12} = \frac{-3 \pm 9}{12}$$
$$\boxed{q_1 = \frac{1}{2}}$$
$$\boxed{\cancel{q_2 = -1}}$$

$$S_5 = ? = 6 \cdot \frac{\left(\frac{1}{2}\right)^5 - 1}{\frac{1}{2} - 1} \qquad S_n = a_1 \cdot \frac{q^n - 1}{q - 1}$$
$$= 6 \cdot \frac{\frac{1}{32} - 1}{\frac{1}{2} - 1} = 6 \cdot \frac{\frac{1}{32} - \frac{32}{32}}{\frac{1}{2} - \frac{2}{2}} =$$
$$= 6 \cdot \frac{-\frac{31}{32}}{-\frac{1}{2}} = 6 \cdot \frac{31 \cdot \cancel{2}}{\cancel{32} \cdot 1} = \overset{3}{\cancel{6}} \cdot \frac{31}{\cancel{16}_{8}} = \boxed{\frac{93}{8} = S_5}$$