Vježbe 9

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ISHOD 3 – integriranje

NEODREĐENI INTEGRAL

$$(\sin x)' = \cos x$$

$$(\sin x + 5)' = \cos x$$

$$\left(\sin x - \frac{10}{3}\right)' = \cos x$$

$$\vdots$$

$$(\sin x + C)' = \cos x$$

obrnuto $\longrightarrow$

NEODREĐENI INTEGRAL

$$\int \cos x \cdot dx = \sin x + C$$

OBLICI KOJI SE SVODE NA POTENCIJU x^n

9.1. Odredite slijedeće neodređene integrale:

Tablični integrali

- $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$

a) $\displaystyle\int x^9 \cdot dx = \frac{x^{10}}{10} + C$

$\displaystyle\int x^{11} \cdot dx = \frac{x^{12}}{12} + C$

b) $\displaystyle\int x \cdot dx = \frac{x^2}{2} + C$

c) $\displaystyle\int e^2 dx = e^2 x + C$

KONSTANTA POMNOŽI X

$\displaystyle\int 5 \cdot dx = 5x + C$

$\displaystyle\int 7 \cdot dx = 7x + C$

$\displaystyle\int dx = \int 1 \cdot dx = 1x + C$

Tablični integrali

- $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq 1$

d) $\displaystyle\int \sqrt{x}\,dx = \int x^{\frac{1}{2}} \cdot dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{x^3} + C$

u potenciju

e) $\displaystyle\int \sqrt[4]{x^3} = \int x^{\frac{3}{4}} \ldots \frac{x^{\frac{7}{4}}}{\ldots} + C = \frac{4}{7}\sqrt[4]{x^7} + C$

u potenciju

e) $\int \sqrt[4]{x^3}\cdot dx = \int x^{\frac{3}{4}}\cdot dx = \frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C = \frac{4}{7}\cdot \sqrt[4]{x^7} + C$

$\boxed{\sqrt[m]{x^n} = x^{\frac{n}{m}}}$

$\frac{3}{4}+1 = \frac{7}{4}$

provjera: $\left(\frac{4}{7}\sqrt[4]{x^7}+C\right)' = \left(\frac{4}{7}\cdot x^{\frac{7}{4}}+C\right)' = \cancel{\frac{4}{7}}\cdot\cancel{\frac{7}{4}}\cdot x^{\frac{7}{4}-1} + 0$

$= x^{\frac{3}{4}} = \sqrt[4]{x^3}$

f) $\int \frac{1}{x^3}\cdot dx = \int x^{-3}\cdot dx = \frac{x^{-2}}{-2} + C = \frac{1}{-2\cdot x^2} + C$

u potenciju $\frac{1}{x^4} = x^{-4}$

$* \int \frac{1}{x^5} dx = \int x^{-5}\cdot dx = \frac{x^{-4}}{-4} + C = \frac{1}{-4\cdot x^4} + C$

$\cancel{\frac{-4}{x^4}}$

g) $\int \frac{1}{\sqrt[3]{x^2}}\, dx = \int x^{-\frac{2}{3}}\cdot dx = \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C = 3\cdot\sqrt[3]{x} + C$

$\frac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$

h) $\int x\sqrt{x}\cdot dx = \int x^{\frac{3}{2}}\cdot dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{5}\cdot\sqrt{x^5} + C$

$x^1\cdot x^{\frac{1}{2}} = x^{1+\frac{1}{2}} = x^{\frac{3}{2}}$

$\boxed{x^m\cdot x^n = x^{m+n}}$

$\boxed{\frac{x^m}{x^n} = x^{m-n}}$

i) $\int \sqrt[3]{\sqrt{x^5}}\cdot dx = \int x^{\frac{5}{6}}\cdot dx = \frac{x^{\frac{11}{6}}}{\frac{11}{6}} + C = \frac{6}{11}\cdot\sqrt[6]{x^{11}} + C$

$\left(\left(x^5\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = x^{\frac{5}{6}}$

$\frac{5}{6}+1 = \frac{11}{6}$

9.2.

Pravila integriranja

$$\int a \cdot f(x)\,dx = a \cdot \int f(x)\,dx$$

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

a) $\int 10x^4 \cdot dx = \cancel{10}^{2} \cdot \frac{x^5}{\cancel{5}} + c = 2x^5 + c$

b) $\int \sqrt{3} \cdot dx = \sqrt{3} \cdot x + C$

c) $\int (x^2 + x) \cdot dx = \frac{x^3}{3} + \frac{x^2}{2} + c$

Tablični integrali

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
- $\int \frac{1}{x}\,dx = \ln|x| + c$

d) $\int \left(\frac{2}{x} - \frac{3}{x^2}\right) dx = \int \left(2 \cdot \frac{1}{x} - 3 \cdot x^{-2}\right) \cdot dx$

$= 2 \cdot \ln|x| - 3 \frac{x^{-1}}{-1} + c$

$= 2\ln|x| + \frac{3}{x} + c$

e) $\int \left(\frac{7}{5}x^6 - \frac{2 \cdot 1}{x} + \frac{1}{\cos^2 \pi}\right) dx = \frac{\cancel{7}}{5} \cdot \frac{x^7}{\cancel{7}} - 2 \cdot \ln|x| + \frac{1}{\cos^2 \pi} x + c$

konstante

$= \frac{1}{5}x^7 - 2\ln|x| + x + c$

$\boxed{\cos\pi = -1}$

f) $\int 2^x (3^x + 5^x)\,dx = \int (2^x \cdot 3^x + 2^x \cdot 5^x) \cdot dx = \int (6^x + 10^x) \cdot dx$

$\boxed{a^n \cdot b^n = (a \cdot b)^n}$

$\boxed{\int a^x\,dx = \frac{a^x}{\ln a} + c}$

$= \frac{6^x}{\ln 6} + \frac{10^x}{\ln 10} + c$

g) $\int (2 + 3^x)^2\,dx = \int (4 + 2 \cdot 2 \cdot 3^x + (3^x)^2) dx$

$(a+b)^2 = a^2 + 2ab + b^2$

$= \int (4 + 4 \cdot 3^x + (3^2)^x) \cdot dx$ $\quad 9^x$

~~12^x~~

$= 4x + 4 \cdot \frac{3^x}{\ln 3} + \frac{9^x}{\ln 9} + c$

Tablični integrali

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
- $\int \frac{1}{x}\,dx = \ln|x| + c$
- $\int e^x\,dx = e^x + c$
- $\int a^x\,dx = \frac{a^x}{\ln a} + c$

$$= 4x + 4\cdot\frac{3^x}{\ln 3} + \frac{9^x}{\ln 9} + c$$

- $\int a^x\,dx = \frac{a^x}{\ln a} + c$
- $\int \sin x\,dx = -\cos x + c$
- $\int \cos x\,dx = \sin x + c$
- $\int \frac{1}{\cos^2 x}\,dx = \operatorname{tg} x + c$
- $\int \frac{1}{\sin^2 x}\,dx = -\operatorname{ctg} x + c$

h) $\int \left(x + \frac{1}{x}\right)^2 dx = \int \left(x^2 + 2\cdot\cancel{x}\cdot\frac{1}{\cancel{x}} + \frac{1}{x^2}\right)\cdot dx$

$$= \int (x^2 + 2 + x^{-2})\,dx$$

$$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + C$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + C$$

9.3.

$$\frac{x^{\frac{1}{2}}}{x^1} = x^{\frac{1}{2}-1} = x^{-\frac{1}{2}}$$

$$\frac{a}{b-1} = \cancel{\frac{a}{b} + \frac{a}{-1}}$$

a) $\int \frac{x + \sqrt{x} - 6}{x} dx = \int \left(\frac{\cancel{x}}{\cancel{x}} + \frac{\sqrt{x}}{x} - \frac{6}{x}\right)\cdot dx$

$$= \int \left(1 + x^{-\frac{1}{2}} - 6\cdot\frac{1}{x}\right)\cdot dx$$

$$= 1x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 6\cdot\ln|x| + C$$

$$= x + 2\sqrt{x} - 6\ln|x| + C$$

$$\frac{1}{2^x} = \frac{1^x}{2^x} = \left(\frac{1}{2}\right)^x$$

b) $\int \frac{10^x - 5^x + 1}{2^x} dx = \int \left(\frac{10^x}{2^x} - \frac{5^x}{2^x} + \frac{1}{2^x}\right)\cdot dx$

- $\boxed{\int a^x\,dx = \frac{a^x}{\ln a} + c}$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$= \int \left(5^x - \left(\frac{5}{2}\right)^x + \left(\frac{1}{2}\right)^x\right) dx$$

$$= \frac{5^x}{\ln 5} - \frac{\left(\frac{5}{2}\right)^x}{\ln\frac{5}{2}} + \frac{\left(\frac{1}{2}\right)^x}{\ln\frac{1}{2}} + C$$

c) $\int \frac{xe^x - 2x + 3}{4x} dx = \int \left(\frac{\cancel{x}e^x}{4\cancel{x}} - \frac{\overset{1}{\cancel{2x}}}{\underset{2}{\cancel{4x}}} + \frac{3}{4x} \right) \cdot dx$

$= \int \left(\frac{1}{4}e^x - \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{x} \right) \cdot dx$

$= \frac{1}{4}e^x - \frac{1}{2}x + \frac{3}{4}\ln|x| + C$

d) $\int \frac{4^x + x^2 2^x - 1}{2^x} dx = \int \left(\frac{4^x}{2^x} + \frac{x^2 \cancel{2^x}}{\cancel{2^x}} - \frac{1}{2^x} \right) dx$

$= \int \left(\left(\frac{4}{2}\right)^x_{=2^x} + x^2 - \left(\frac{1}{2}\right)^x \right) \cdot dx$

$= \frac{2^x}{\ln 2} + \frac{x^3}{3} - \frac{\left(\frac{1}{2}\right)^x}{\ln \frac{1}{2}} + C$

e) $\int \frac{\sin^3 x + \cos^2 x}{\sin^2 x} dx = \int \left(\frac{\sin^{\cancel{3}} x}{\cancel{\sin^2 x}} + \frac{\cos^2 x}{\sin^2 x} \right) \cdot dx$

nema u tablici
pa mijenjamo $\cos^2 x = 1 - \sin^2 x$

$= \int \left(\sin x + \frac{1 - \sin^2 x}{\sin^2 x} \right) \cdot dx$

$= \int \left(\sin x + \frac{1}{\sin^2 x} - \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}}_{1} \right) \cdot dx$

$= -\cos x - \operatorname{ctg} x - 1x + C$

f) $\int \frac{3e^{2x} - 2e^x + 1}{e^x} dx = \int \left(\frac{3e^{2x}}{e^x} - \frac{2\cancel{e^x}}{\cancel{e^x}} + \frac{1}{e^x} \right) \cdot dx = \int \left(3e^x - 2 + \left(\frac{1}{e}\right)^x \right) \cdot dx$

$e^{2x-x} = e^x$

$= 3e^x - 2x + \frac{\left(\frac{1}{e}\right)^x}{\ln \frac{1}{e}} + C$

$\ln \frac{1}{e} = -1$

$= 3e^x - 2x - \left(\frac{1}{e}\right)^x + C$

$\ln \frac{1}{e} = \ln e^{-1} = -1 \cdot \underbrace{\ln e}_{1} = -1$

Tablični integrali

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
- $\int \frac{1}{x} \, dx = \ln|x| + c$
- $\int e^x \, dx = e^x + c$
- $\int a^x \, dx = \frac{a^x}{\ln a} + c$
- $\int \sin x \, dx = -\cos x + c$
- $\int \cos x \, dx = \sin x + c$
- $\int \frac{1}{\cos^2 x} \, dx = \operatorname{tg} x + c$
- $\int \frac{1}{\sin^2 x} \, dx = -\operatorname{ctg} x + c$

Trigonometrijske funkcije

$\operatorname{tg} x = \frac{\sin x}{\cos x}, \quad \operatorname{ctg} x = \frac{\cos x}{\sin x}$

$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x}, \quad \boxed{\cos^2 x + \sin^2 x = 1}$

- $\boxed{\int a^x \, dx = \frac{a^x}{\ln a} + c}$