METODA SUPSTITUCIJE

Ponovimo:

1) $\int x^7 \cdot dx = \frac{x^8}{8} + C$

2) $\int \sqrt{x} \cdot dx = \int x^{\frac{1}{2}} \cdot dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3} \cdot \sqrt{x^3} + C$

$\vdots$

10.1. METODA SUPSTITUCIJE

ZAMJENA

vratiti x-eve

a) $\int \sqrt{3+x}\, dx = \left| \begin{array}{l} 3+x = t \;/' \\ 1 \cdot dx = 1 \cdot dt \end{array} \right| = \int \sqrt{t} \cdot dt = \int t^{\frac{1}{2}} \cdot dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3} \cdot \sqrt{t^3} + C$

$= \frac{2}{3} \sqrt{(3+x)^3} + C$

ZAMJENA

b) $\int \frac{1}{\sqrt[3]{(2-3x)^2}}\, dx = \left| \begin{array}{l} 2-3x = t \;/' \\ -3 \cdot dx = 1 \cdot dt \;/:(-3) \quad dx = ? \\ dx = -\frac{1}{3} dt \end{array} \right| = \int \frac{1}{\sqrt[3]{t^2}} \cdot \left(-\frac{1}{3} dt\right) = -\frac{1}{3} \cdot \int t^{-\frac{2}{3}} \cdot dt$

$ax + b = t$
LINEARNI IZRAZ

u potenciju $\frac{1}{\sqrt[3]{t^2}} = \frac{1}{t^{\frac{2}{3}}} = t^{-\frac{2}{3}}$

$= -\frac{1}{3} \cdot \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + C = -\sqrt[3]{t} + C = -\sqrt[3]{2-3x} + C$

MALO SLOŽENIJE: $\boxed{\text{ZA } t \text{ UZIMAMO IZRAZ ČIJA DERIVACIJA MNOŽI } dx}$

c) $\int \frac{x\, dx}{1+x^2} = \left| \begin{array}{l} 1+x^2 = t \;/' \\ 2x \cdot dx = dt \;/:2x \\ dx = \frac{dt}{2x} \end{array} \right| = \int \frac{\cancel{x}}{t} \cdot \frac{dt}{2\cancel{x}} = \frac{1}{2} \cdot \int \frac{1}{t} dt = \frac{1}{2} \cdot \ln|t| + C$

$\boxed{\int \frac{1}{x} \cdot dx = \ln|x| + C}$

$= \frac{1}{2} \ln|1+x^2| + C$

d) $\int \frac{x^2}{\sqrt{1+x^3}}\, dx = \left| \begin{array}{l} 1+x^3 = t \;/' \\ 3x^2 \cdot dx = dt \;/:(3x^2) \\ dx = \frac{dt}{3x^2} \end{array} \right| = \int \frac{\cancel{x^2}}{\sqrt{t}} \cdot \frac{1 \, dt}{3\cancel{x^2}} = \frac{1}{3} \int t^{-\frac{1}{2}} \cdot dt = \frac{1}{3} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$

u potenciju

$$= \frac{1}{3}\cdot 2\cdot\sqrt{t} + C = \frac{2}{3}\sqrt{1+x^3} + C$$

e) $\int \frac{2x+1}{x^2+x-3}\boxed{dx} = \left| \begin{array}{l} x^2+x-3 = t \;/\,' \\ (2x+1)\cdot dx = dt \\ \boxed{dx} = \boxed{\frac{dt}{2x+1}} \end{array} \right| = \int \frac{\cancel{2x+1}}{t}\cdot\boxed{\frac{dt}{\cancel{2x+1}}} = \int \frac{1}{t}\cdot dt$

$\int \frac{1}{x}\cdot dx = \ln|x|$

$$= \ln|t| + C = \ln|x^2+x-3| + C$$

f) $\int 3x\,e^{x^2-1}\boxed{dx} = \left| \begin{array}{l} x^2-1 = t \;|\,' \\ 2x\cdot dx = 1\cdot dt \;|:2x \\ \boxed{dx} = \boxed{\frac{dt}{2x}} \end{array} \right| = \int 3\cancel{x}\cdot e^{t}\cdot\boxed{\frac{dt}{2\cancel{x}}} = \frac{3}{2}\cdot\int e^t\cdot dt = \frac{3}{2}e^{t} + C$

$\int e^x dx = e^x + c$

$$= \frac{3}{2}e^{x^2-1} + C$$

10.2.

ili $\ln^2 x$

a) $\int \frac{1\cdot\ln x}{x}\boxed{dx} = \left| \begin{array}{l} \ln x = t \;|\,' \\ \frac{1}{x}\cdot dx = dt \;/\cdot x \\ \boxed{dx} = \boxed{x\cdot dt} \end{array} \right| = \int \frac{t}{\cancel{x}}\cdot\boxed{\cancel{x}\cdot dt} = \int t^1\cdot dt = \frac{t^2}{2} + C = \frac{(\ln x)^2}{2} + C$

$\int x^n\cdot dx = \frac{x^{n+1}}{n+1}$

b) $\int \frac{1\cdot\boxed{dx}}{x\ln^2 x}$ [redacted] $= \left| \begin{array}{l} \ln x = t \;|\,' \\ \frac{1}{x}\cdot dx = dt \;|\cdot x \\ \boxed{dx} = \boxed{x\cdot dt} \end{array} \right| = \int \frac{1}{\cancel{x}\cdot t^2}\cdot\boxed{\cancel{x}\cdot dt} = \int t^{-2}\cdot dt = \frac{t^{-1}}{-1} + C$

POTENCIJA

$$= -\frac{1}{t} + C = -\frac{1}{\ln x} + C$$

potencije $\quad \sqrt[m]{x^n} = x^{\frac{n}{m}}$

c) $\int \frac{\sqrt{1+\ln x}}{x}\boxed{dx} = \left| \begin{array}{l} 1+\ln x = t \;/\,' \\ \frac{1}{x}\cdot dx = dt \;/\cdot x \\ \boxed{dx} = \boxed{x\cdot dt} \end{array} \right| = \int \frac{\sqrt{t}}{\cancel{x}}\cdot\boxed{\cancel{x}\cdot dt} = \int t^{\frac{1}{2}}\cdot dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$

$$= \frac{2}{3}\cdot\sqrt{t^3}+c = \frac{2}{3}\cdot\sqrt{(1+\ln x)^3}+c$$

d) $$\int \frac{e^x}{e^x+1}\,dx = \left| \begin{array}{l} e^x+1=t \;|' \\ e^x\cdot dx = dt \;/ : e^x \\ dx = \frac{dt}{e^x} \end{array} \right| = \int \frac{\cancel{e^x}}{t}\cdot\frac{dt}{\cancel{e^x}} = \int \frac{1}{t}\cdot dt = \ln|t|+c$$

$$\int \frac{1}{x}\cdot dx = \ln|x|$$

$$= \ln|e^x+1|+c$$

$$(\quad)$$

f) $$\int e^x\sqrt{2-e^x}\,dx = \left| \begin{array}{l} 2-e^x = t \;|' \\ -e^x\cdot dx = dt \\ dx = \frac{dt}{-e^x} \end{array} \right| = \int \cancel{e^x}\cdot\sqrt{t}\cdot\frac{dt}{-\cancel{e^x}} = -\int t^{\frac{1}{2}}\cdot dt = -\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+c$$

$$= -\frac{2}{3}\sqrt{t^3}+c = -\frac{2}{3}\sqrt{(2-e^x)^3}+c$$

$$(e^{\square})' = e^{\square}\cdot \square'$$

e) $$\int \frac{e^{2x}}{1-3e^{2x}}\,dx = \left| \begin{array}{l} 1-3e^{2x} = t \;|' \\ -3\cdot e^{2x}\cdot 2\cdot dx = dt \\ dx = \frac{dt}{-6e^{2x}} \end{array} \right| = \int \frac{\cancel{e^{2x}}}{t}\cdot\frac{dt}{-6\cancel{e^{2x}}} = -\frac{1}{6}\int\frac{1}{t}\cdot dt$$

$$= -\frac{1}{6}\ln|t|+c = -\frac{1}{6}\ln|1-3e^{2x}|+c$$

$$\frac{4}{3}\cdot(-\cos t) \checkmark$$

$$\cancel{\frac{4}{3}-\cos t}$$

10.3

linearni izraz $ax+b=t$

a) $$\int 4\sin(3x)\,dx = \left| \begin{array}{l} 3x = t \;|' \\ 3\cdot dx = dt \\ dx = \frac{dt}{3} \end{array} \right| = \int 4\cdot\sin t\cdot\frac{dt}{3} = \frac{4}{3}\int \sin t\cdot dt = -\frac{4}{3}\cos t + c$$

$$\int \sin x\cdot dx = -\cos x$$

$$= -\frac{4}{3}\cos 3x + c$$

b) $$\int \cos x\sin^2 x\,dx = \left| \begin{array}{l} \sin x = t \;|' \\ \cos x\cdot dx = dt \;/ : \cos x \end{array} \right| = \int \cancel{\cos x}\cdot t^2\cdot\frac{dt}{\cancel{\cos x}} = \int t^2\cdot dt = \frac{t^3}{3}+c$$

$$= \sin^3 x$$

$$\cos x \cdot dx = dt \,/ : \cos x \qquad dx = \frac{dt}{\cos x}$$

$$= \frac{\sin^3 x}{3} + C$$

c) $$\int \frac{\cos x}{\sin^4 x} dx = \left| \begin{array}{l} \sin x = t \;/' \\ \cos x \cdot dx = dt \\ dx = \frac{dt}{\cos x} \end{array} \right| = \int \frac{\cancel{\cos x}}{t^4} \cdot \frac{dt}{\cancel{\cos x}} = \int \frac{1}{t^4} dt = \int t^{-4} dt = \frac{t^{-3}}{-3} + C$$

$$= \frac{1}{-3 \cdot t^3} + C = \frac{1}{-3 \sin^3 x} + C$$

d) $$\int \frac{\sin x}{1 + 2\cos x} dx = \left| \begin{array}{l} 1 + 2\cos x = t \,/' \\ -2\sin x \cdot dx = dt \,/ : (-2\sin x) \\ dx = \frac{dt}{-2\sin x} \end{array} \right| = \int \frac{\cancel{\sin x}}{t} \cdot \frac{dt}{-2\cancel{\sin x}} = -\frac{1}{2} \int \frac{1}{t} \cdot dt$$

$$= -\frac{1}{2} \ln|t| + C = -\frac{1}{2} \ln|1 + 2\cos x| + C$$

e) $$\int \operatorname{ctg} x \, dx = \int \frac{\cos x}{\sin x} \cdot dx = \left| \begin{array}{l} \sin x = t \,/' \\ \cos x \cdot dx = dt \\ dx = \frac{dt}{\cos x} \end{array} \right| = \int \frac{1 \cdot \cancel{\cos x}}{t} \cdot \frac{dt}{\cancel{\cos x}} = \ln|t| + C$$

$$\boxed{\operatorname{ctg} x = \frac{\cos x}{\sin x}} \qquad = \ln|\sin x| + C$$

$$(\cos \square)' = -\sin \square \cdot \square'$$

f) $$\int \frac{\sin(\ln x)}{x \cos^3(\ln x)} dx = \left| \begin{array}{l} \cos(\ln x) = t \,/' \\ -\sin(\ln x) \cdot \frac{1}{x} \cdot dx = dt \,/ \cdot \frac{x}{-\sin(\ln x)} \\ dx = \frac{x}{-\sin(\ln x)} \cdot dt \end{array} \right| = \int \frac{\cancel{\sin(\ln x)}}{\cancel{x} \cdot t^3} \cdot \boxed{\frac{\cancel{x}}{-\cancel{\sin(\ln x)}} \cdot dt}$$

$$= -\int t^{-3} \cdot dt = -\frac{t^{-2}}{-2} + C = \frac{1}{2 \cdot t^2} + C = \frac{1}{2\cos^2(\ln x)} + C$$