

Geometrija u prostoru

13.1

a) $A(1, -2)$

$B(-3, 1)$

$$\vec{r} = \vec{r}_A + t \cdot \vec{s}$$

$$\vec{r} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} + t \cdot \begin{bmatrix} -4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 - 4t \\ -2 + 3t \end{bmatrix} \quad t \in \mathbb{R}$$

$$\vec{s} = \vec{AB} = -4\vec{i} + 3\vec{j} \quad (\text{dolje } x \text{ od gornjeg osu})$$

b) $y = 2x - 1 \rightarrow$ odaberemo bilo koji x , da dobijemo točku npr $x = 0$

$$\vec{r} = \vec{r}_0 + t \cdot \vec{s} \quad \vec{s} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + t \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad y = 2 \cdot 0 - 1$$

$$y = -1$$

$$\vec{r} = \begin{bmatrix} 0 & t \\ -1 & 2t \end{bmatrix} = \begin{bmatrix} t \\ -1 + 2t \end{bmatrix} \quad A(0, -1) \quad t \in \mathbb{R}$$

c)

$A(3, 1)$

$2y - x + 3 = 0$

$y = \frac{1}{2}x - \frac{3}{2}$

$s_x = \frac{1}{2} \rightarrow \vec{s} = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$

$$\vec{r} = \vec{r}_A + t \cdot \vec{s}$$

$$\vec{r} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 3 + t \\ 1 + \frac{1}{2}t \end{bmatrix}$$

paralelni pravci
imaju isti smjer.

Konvergenz der Reihe

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ -3 & 0 & -1 & 2 \\ 2 & 0 & 0 & -6 \\ -1 & 1 & 2 & 1 \end{bmatrix} = 2 \cdot (-1) = -2$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{bmatrix} = 1 + 2 - (4) = -1$$

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$$y = x = f(x)$$

$$\vec{r} = \begin{bmatrix} x \\ f(x) \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ x \end{bmatrix}$$

$$w) \alpha = 90^\circ \quad \text{Ind} = 60^\circ \quad \alpha = 40^\circ$$

$$c) \vec{r} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \cdot \vec{r}$$

$$\vec{r} = \begin{bmatrix} \cos 40^\circ & -\sin 40^\circ \\ \sin 40^\circ & \cos 40^\circ \end{bmatrix} \cdot \begin{bmatrix} x \\ x \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 0.7660 & -0.6428 \\ 0.6428 & 0.7666 \end{bmatrix} \cdot \begin{bmatrix} x \\ x \end{bmatrix}$$

$$= \begin{bmatrix} 0.7660x - 0.6428x \\ 0.6428x + 0.7660x \end{bmatrix}$$

13.10

$$\vec{r} = \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$$

a) $S_x = 2$

1. row

x se mijerun
y isth

$$\vec{r}' = \begin{bmatrix} 2+2t \\ 2-t \end{bmatrix}$$

2. row

$$\vec{r}'' = S_x \cdot \vec{r} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$$

$$= \begin{bmatrix} 2+2t \\ 2-t \end{bmatrix}$$

b) $S_y = \frac{1}{3}$

1. row x isth

y se mijerun

$$= \begin{bmatrix} 1+t \\ \frac{2}{3} - \frac{1}{3}t \end{bmatrix}$$

2 row

$$\vec{r}''' = S_y \cdot \vec{r} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$$

$$= \begin{bmatrix} 1+t \\ \frac{2}{3} - \frac{1}{3}t \end{bmatrix}$$

mpk. $t=0, t=1$

$$1+0 = 1 \quad \text{1. titik } A(1, 2)$$

$$2-0 = 2$$

$$1+1 = 2 \quad \text{2. titik } B(2, 1)$$

$$2-1 = 1$$

$\vec{A}(2, 2) \quad \vec{B}(4, 1)$

13.11

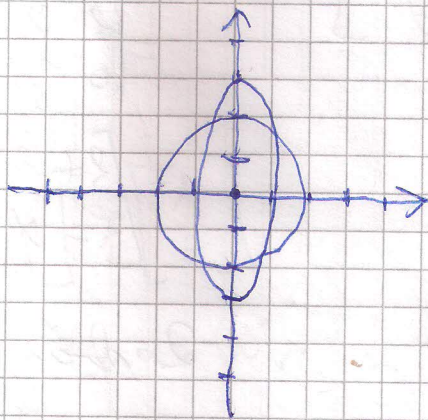
apert: $(x-p)^2 + (y-q)^2 = R^2$

$S(p, q)$

$S(0,0), R=2$

$x^2 + y^2 = 4$

vektorform: $\vec{r} = \begin{bmatrix} R \cos t \\ R \sin t \end{bmatrix} \rightarrow \begin{bmatrix} 2 \cos t \\ 2 \sin t \end{bmatrix}$



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a=1, b=\sqrt{3}$

k_x
 $2 \rightarrow 1$

$2 \cdot k_x = 1/2$
 $2x = \frac{1}{2}$

k_y
 $2 \rightarrow 3$

$2 \cdot k_y = 3/2$
 $k_y = \frac{3}{2} = 1.5$

19.1

a) $A(1, -2, 2)$

$B(0, -3, 1)$

$\vec{s} = \vec{AB}$

B immer A

$\vec{r} = \vec{r}_A + t \cdot \vec{s}$

$\vec{r} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + t \cdot \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

$\vec{r} = \begin{bmatrix} 1-t \\ -2-t \\ 2-t \end{bmatrix}$

$$b) \quad x \begin{pmatrix} -2 \\ 3 \end{pmatrix} = y \begin{pmatrix} 1 \\ 0 \end{pmatrix} = z \begin{pmatrix} 2 \\ -1 \end{pmatrix} \rightarrow \text{singler}$$

$$\frac{x - x_A}{s_x} = \frac{y - y_A}{s_y} = \frac{z - z_A}{s_z}$$

$$\vec{s} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, A(2, -1, 0) \rightarrow \text{demi produkt}$$

$$\vec{r} = \vec{r}_A + \lambda \cdot \vec{s}$$

$$\vec{r} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \lambda \cdot \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 2 + 3\lambda \\ -1 \\ -\lambda \end{bmatrix}, \lambda \in \mathbb{R}$$

c)

$$A(3, -2, 1)$$

$$\vec{r} = \begin{bmatrix} 2 - \lambda \\ 1 - 3\lambda \\ -2 \end{bmatrix}$$

parallelni povi možni isto jednake ili proporcionalne
vektore smjern \vec{s} .

-nema su jednaki:

$$\vec{s} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\vec{r}' = \vec{r}_A + \lambda \cdot \vec{s}$$

$$\vec{r}' = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + \lambda \cdot \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\vec{r}' = \begin{bmatrix} 3 - \lambda \\ -2 + 3\lambda \\ 1 \end{bmatrix}$$

14.2

a)

$$A(3, -1, 0)$$

$$B(-1, -2, 2)$$

$$\frac{x-3}{-4} = \frac{y+1}{-1} = \frac{z}{2}$$

$$\vec{s} = \vec{AB}$$

$$\vec{s} = \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}$$

b)

$$\vec{r} = \begin{bmatrix} -t \\ 2 \\ 1-3t \end{bmatrix}$$

1. rovin

$$\vec{s} = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$$

$$A(0, 2, 1)$$

... opet tu gora

14.3

a)

$$\vec{r}_1 = \begin{bmatrix} 2-3t \\ 2+t \\ 3 \end{bmatrix}$$

Rijednacima

$$\vec{r}_2 = \begin{bmatrix} 4-4t \\ 4 \\ -1+2t \end{bmatrix}$$

$$2-3 \cdot \overset{t_1}{2} = 4-4 \cdot \overset{t_2}{2}$$

$$\boxed{-4 = -4} \quad \checkmark$$

$$\begin{cases} 2-3t_1 = 4-4t_2 \\ 2+t_1 = 4 \\ 3 = -1+2t_2 \end{cases}$$

$$2+t_1 = 4$$

preseri
so zjed

$$\boxed{t_1 = 2}$$

u(-4, 4)

$$3 = -1+2t_2$$

$$t_2 = 2$$

uvrstin

u t₂ u t₁

u t₁ u t₂

$$2-3 \cdot 2 = -4$$

li)

$$\frac{x}{3} = \frac{y-5}{-2} = \frac{z-2}{1}$$

$$A_1(0, 5, 2)$$

$$\vec{s}_1 = (3, -2, 1)$$

$$\frac{x-1}{2} = \frac{y+2}{1} = \frac{z+1}{1}$$

$$A_2(1, 2, -1)$$

$$\vec{s}_2 = (2, 1, 1)$$

$$\vec{r}_1 = \begin{bmatrix} 3t \\ 5-2t \\ 2+t \end{bmatrix} \quad \vec{r}_2 = \begin{bmatrix} 1+2k \\ 2+k \\ -1+k \end{bmatrix}$$

$$\begin{cases} 3t_1 = 1+2k_2 \\ 5-2t_1 = 2+k_2 \\ 2+t_1 = -1+k_2 \end{cases}$$

$$2+t_1 = -1+k_2$$

$$A_1 = \begin{matrix} \text{---} \\ -3+k_2 \end{matrix}$$

$$t_1 = -3 + 10$$

$$t_1 = 7$$

$$3t_1 = 1+2k_2$$

$$3(-3+k_2) = 1+2k_2$$

$$-9+3k_2 = 1+2k_2$$

$$k_2 = 10$$

prüfen

$$5-2t_1 = 2+k_2$$

$$5-2 \cdot 7 = 2+10$$

$$-9 = 12$$

ne stimmt

14.4

$$a) \vec{r}_1 = \begin{bmatrix} 2-x \\ 2+x \\ 3 \end{bmatrix} \quad \vec{r}_2 = \begin{bmatrix} 7-x \\ 2x \\ -1+3x \end{bmatrix}$$

normen an drücken: $\vec{s}_1 \cdot \vec{s}_2 = 0$

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -x \\ 2 \\ 3x \end{bmatrix} = 0$$

$$-1 \cdot (-x) + 1 \cdot 2 + 0 \cdot 3x = 0$$

$$x + 2 = 0$$

$$x = -2$$

prüfen

$$\vec{s}_1 \perp \vec{s}_2$$

minimale su

13.2

a) $A(3,0)$

$B(-2,2)$

$$\vec{r} = \vec{r}_A + t \cdot \vec{d}$$

$$\vec{r} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 - 5t \\ 0 + 2t \end{bmatrix} = \begin{bmatrix} 3 - 5t \\ 2t \end{bmatrix}$$

$$\vec{d} = \vec{AB} = -5\vec{i} + 2\vec{j} \quad t \in [0, 1]$$

b) $A(2, -1)$

$B(0, 4)$

$$\vec{r} = \vec{r}_A + t \cdot \vec{d}$$

$$\vec{r} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} + t \cdot \begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 - 2t \\ -1 + 5t \end{bmatrix}$$

$$\vec{d} = \vec{AB} = -2\vec{i} + 5\vec{j}$$

$t \in [0, 1]$

13.3

a) $\vec{r} = \begin{bmatrix} 1 - 2t \\ -3t \end{bmatrix}$

$t=0$

$A(1, 0)$

$t=1$

$B(-1, -3)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$y - 0 = \frac{-3 - 0}{-1 - 1} \cdot (x - 1)$$

$$y - 0 = \frac{-3}{-2} \cdot (x - 1)$$

$$y - 0 = \frac{-3}{-2} x + \frac{3}{2}$$

$$y = \frac{-3}{-2} x + \frac{3}{2}$$

$$y = \frac{3}{2} x + \frac{3}{2}$$

b)

$$\vec{r} = \begin{bmatrix} 2+t \\ 1-2t \end{bmatrix}$$

$$A(2, 1) \quad t=0$$

$$B(3, -1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$y - 1 = \frac{-1 - 1}{3 - 2} \cdot (x - 2)$$

$$y - 1 = \frac{-2}{1} \cdot (x - 2)$$

$$y - 1 = -2x + 4$$

$$y = -2x + 5$$

13.4

translacija

$$t \in [0, 1]$$

$$\vec{r}_1 = \vec{r} + t \vec{d}$$

$$\vec{r} = \begin{bmatrix} 1-2t \\ 2+t \end{bmatrix} \xrightarrow{t=0} A(1, 2)$$

$$\vec{r} = \begin{bmatrix} 1-2t \\ 2+t \end{bmatrix}$$

$$\xrightarrow{t=1} B(-1, 3)$$

$$\vec{d} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

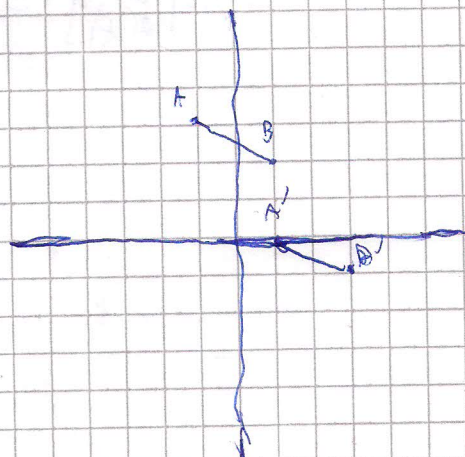
→ pomjereno x-ovu
→ pomjereno y-ovu

$$\vec{r}_1 = \vec{r} + t \vec{d}$$

$$\vec{r}_1 = \begin{bmatrix} 1-2t \\ 2+t \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$\vec{r}_1 = \begin{bmatrix} 3-2t \\ -1+t \end{bmatrix} \xrightarrow{t=0} A'(3, -1)$$

$$\xrightarrow{t=1} B'(1, 0)$$



13.5

Inversio

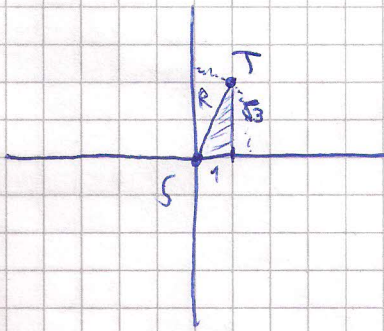
$$\vec{r} = \begin{bmatrix} R \cos t \\ R \sin t \end{bmatrix}$$

$$r(1, \sqrt{3})$$

$$\vec{r} = \begin{bmatrix} 2 \cos t \\ 2 \sin t \end{bmatrix}$$

$$\vec{r}_1 = \begin{bmatrix} 2 \cos t \\ 2 \sin t \end{bmatrix} + \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

$$\vec{r}_1 = \begin{bmatrix} 1 + 2 \cos t \\ \sqrt{3} + 2 \sin t \end{bmatrix}$$



$$R^2 = 1^2 + (\sqrt{3})^2$$

$$R = 2$$

$$\vec{r} = S \vec{T} \quad S(0,0) \rightarrow$$

$$\vec{r} = 1 \vec{i} + \sqrt{3} \vec{j} \rightarrow T(1, \sqrt{3})$$

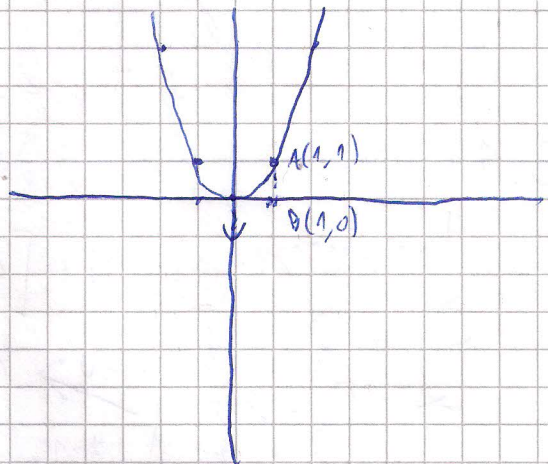
$$\vec{r} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

13.6

$$y = x^2 = f(x)$$

$$\vec{r} = \begin{bmatrix} x \\ f(x) \end{bmatrix} = \begin{bmatrix} x \\ x^2 \end{bmatrix}$$

$$y = x^2$$



multid

$$x_{1,2} = \dots$$

$$x_{1,2} = 0$$

$$\vec{r}_1 = \begin{bmatrix} x \\ x^2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\vec{r}_1 = \begin{bmatrix} x \\ x^2 - 1 \end{bmatrix}$$

x	-2	-1	1	2
y	4	1	1	4

13.7

$$\vec{r} = \begin{bmatrix} -t \\ 3-2t \end{bmatrix}$$

predloženo je poberi:

1) x-osi : x ista
y mijenja predznak

2) y-osi : x mijenja predznak
y ista

$$\vec{r}' = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

2. način

$$\vec{r}' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -t \\ 3-2t \end{bmatrix}$$

$$\vec{r}' = \begin{bmatrix} 1(-t) - 0(3-2t) \\ 0(-t) - 1(3-2t) \end{bmatrix}$$

$$\vec{r}' = \begin{bmatrix} -t \\ -3+2t \end{bmatrix}$$

13.8

$$S(1, 1)$$

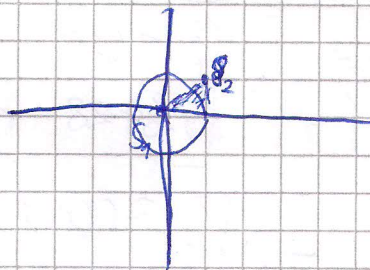
$$R=1$$

$$\vec{r}_x = \begin{bmatrix} 1 \cos t \\ 1 \sin t \end{bmatrix} = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

$$\vec{r}' = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{r}' = \begin{bmatrix} 1 + \cos t \\ 1 + \sin t \end{bmatrix}$$

$$\vec{r}'_2 = \begin{bmatrix} -1 - \cos t \\ 1 + \sin t \end{bmatrix} \rightarrow \text{normal} \quad \text{mjerosnog}$$



$$\vec{r} = S \vec{e}$$

$$\vec{r} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2.) prolehi

$$\vec{s}_1 \in S_2$$

$$\vec{s}_1 = \lambda \cdot \vec{s}_2$$

određi broj

$$\vec{s}_1 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \quad \vec{s}_2 = \begin{bmatrix} -6 \\ 2\alpha \\ -2 \end{bmatrix}$$

0 broj broj

može biti da deluje?

$$\lambda = (-2)$$

$$-2 \cdot \lambda = 2\alpha$$

$$-2 \cdot (-2) = 2\alpha$$

$$\alpha = 2$$

Primer ispit

Tablica sinusa i kosinusa neka !!! (V predložiti ma)

$$1. \quad 2A^T + \frac{1}{2}X = B \cdot C$$

$$\frac{1}{2}X = B \cdot C - 2A^T$$

Sve rjesiš i nekog parametra = 2 da se deluje X.

2.

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & 1 & 0 & -3 \\ -1 & -3 & 1 & -1 \end{array} \right] \begin{array}{l} / \cdot (-2) \\ / \cdot 1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 5 & -2 & -1 \\ 0 & -5 & 2 & -5 \end{array} \right] / : 5$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & -5 & 2 & -5 \end{array} \right]$$

(Može se pronaći jedinstven parametar (*) !!!)

3.

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

metoda Gauss

4.

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & -1 & -1 & 1 \\ 2 & 1 & -2 & 0 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

U besudu se unjela matrica 4×4 matrica, dakle Laplace razvij unjela.

5.

$$A \cdot X + B \cdot X = 2I$$

$$(A+B)^{-1} \cdot (A+B) \cdot X = 2I$$

$$X = (A+B)^{-1} \cdot 2I$$

$$X = 2 \cdot (A+B)^{-1}$$

izlučivanje

$$A+B = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}$$

$$A+B = \begin{vmatrix} 3 & -3 \\ 1 & 4 \end{vmatrix}$$

$$(A+B)^{-1} = \frac{1}{A+B} \dots$$

6.

a) $\vec{B} = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$...

c) i d) jedan način zbog 4. metoda.

4. je težina !!!
ishod

~ Koliko je lako izračunati matricu? (Može doći kao pitanje tu)

- 2. Matrica
- 2. Gauss
- 1. Determinante i Matrica...

Iskusi na 3. ishodu

4. "redni"

1.

2.

$R=3$ $S(0,0)$ pomen $\begin{bmatrix} 3 \cos t \\ 3 \sin t \end{bmatrix}$ $r' = r + t$

$\begin{bmatrix} 9 + 3 \cos t \\ -1 + 3 \sin t \end{bmatrix}$

3. *

→ om je teži primjer, dajem su pravi konkretni brojevi

$R = \begin{vmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{vmatrix} = \begin{vmatrix} \dots & \dots \\ \dots & \dots \end{vmatrix} \cdot \begin{vmatrix} 9 + 3t \\ -1 + 3t \end{vmatrix} = \text{rijeka}$

→ skica: postavi (*) na odru usjednost i dječnjak od usjednost i rotiraj.

4.

$S = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$y = x^3$

$\vec{r} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x^3 \end{bmatrix} = \begin{bmatrix} 2x \\ x^3 \end{bmatrix}$

skica = original nos skolinova

Kise elci i to testu da skolinove ne bude direktno skolinova nego mi moramo skoditi koliku trebam skolinoti.

5.

$$\vec{r} = \vec{r} + \vec{r} + \vec{r}$$

$$x \quad y \quad z$$

$$\vec{r} = \begin{bmatrix} x + xV + xU \\ y + yV + yU \\ z + zV + zU \end{bmatrix}$$

↓
negled
mesenje
vsega.

6.

$$\begin{aligned} x - 2y - z + 3 &= 0 \\ -x + y + 2z - 1 &= 0 \end{aligned} \rightarrow \text{Gauss}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ -1 & 1 & 2 & 1 \end{array} \right]$$

vektorski oblik $\vec{r} = \begin{bmatrix} 3x+1 \\ x+2 \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

$$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z}{1}$$

kananski oblik

7. → tu največ hude - standardno rešo pri

$$\vec{r} = \begin{bmatrix} 1-x \\ -1-x \\ 2x \end{bmatrix} \quad -x + 2y - z + 2 = 0$$

$$\begin{cases} x = 1-x \\ y = -1-x \\ z = 2x \end{cases}$$

$$\boxed{x = \frac{1}{3}}$$

na x
delimo
za tri
razisa
yena

4. rind

1.

2.

$R=3$ $\hookrightarrow (0,0)$ gora $\begin{bmatrix} 3 \cos t \\ 3 \sin t \end{bmatrix}$ $r' = r + t$

$$\begin{bmatrix} 9 + 3 \cos t \\ -1 + 3 \sin t \end{bmatrix}$$

3.*

\rightarrow ono je teži primjer, daju su pravi konkretni brojevi

$R = \begin{vmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{vmatrix} = \begin{vmatrix} \dots & \dots \\ \dots & \dots \end{vmatrix} \cdot \begin{vmatrix} 1 + 50t \\ -1 + t \end{vmatrix} = \text{rijes}$

\rightarrow skica: postavi (*) su neki injeđnost i dajući od vektoru i rotirajući.

4.

$$S = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$y = x^3$$

$$\vec{r} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x^3 \end{bmatrix} = \begin{bmatrix} 2x \\ x^3 \end{bmatrix}$$

skica = original no skicirano

Kada svi ti su testu da skicirajući se bude direktno skicirano međa mi moramo skicirati koliku trebamo skicirati.

5.

$$\vec{r} = \vec{r}_x + \vec{r}_y + \vec{r}_z$$

$$\vec{r} = \begin{bmatrix} x + xV + xU \\ y + yV + yU \\ z + zV + zU \end{bmatrix}$$

↓
nigled
mješavina
vješta.

6.

$$\begin{aligned} x - 2y - z + 3 &= 0 \\ -x + y + 2z - 1 &= 0 \end{aligned} \rightarrow \text{Gauss}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ -1 & 1 & 2 & 1 \end{array} \right]$$

vertikalni oblik $\vec{r} = \begin{bmatrix} 3x+1 \\ x+2 \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

$$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z}{1}$$

horizontalni oblik

7. → tu uvijek bude - standard nebo pret

$$\vec{r} = \begin{bmatrix} 1-t \\ -1-t \\ 2t \end{bmatrix} \quad -x + 2y - z + 2 = 0$$

$$\begin{cases} x = 1-t \\ y = -1-t \\ z = 2t \end{cases}$$

$$t = \frac{1}{3}$$

ako se
odluže
pa tri
razisa
yasa

8. Za jedne ravniše simetrike srua tuđu

Delit sruitudu (uzel srua x, delit y, z) -

$$x=0 \quad T(0,0,3)$$

$$y=0$$

$z=3$ od formule

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

vjele

14.4

14.4

c)

$$\vec{n}_1 = \begin{vmatrix} 1+2x \\ x \\ -1-x \end{vmatrix}$$

$$\frac{x-3}{-1} = \frac{y-0}{2} = \frac{z+1}{-1}$$

$$\vec{n}_2 = \begin{vmatrix} 3-x \\ x+2x \\ -1-x \end{vmatrix}$$

$$1+2x = 3-u$$

$$x = u + 2u$$

$$-1-x = -1-u$$

$$-1-x = -1-u$$

$$-x = -u$$

$$x = u$$

$$1+2x = 3-u$$

$$1+2u = 3-u$$

$$u = \frac{2}{3}$$

$$x = \frac{2}{3}$$

$$x = u + 2u$$

$$\frac{2}{3} = u + 2 \cdot \frac{2}{3}$$

$$u = -\frac{2}{3}$$

14.5

$$\vec{n}_1 = \begin{vmatrix} 2-x \\ 3 \\ 2x \end{vmatrix}$$

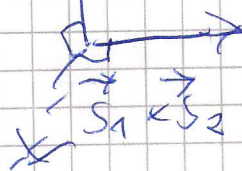
$$S_1 = \begin{vmatrix} -1 \\ 0 \\ 2 \end{vmatrix}$$

$$\vec{n}_2 = \begin{vmatrix} 1+3x \\ 3-x \\ 2 \end{vmatrix}$$

$$S_2 = \begin{vmatrix} 3 \\ -1 \\ 0 \end{vmatrix}$$

vektor skruit na dnu srua

$\vec{S}_1 \wedge \vec{S}_2$ je vektor $S_1 \times S_2$



$$\vec{s}_1 \times \vec{s}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ -1 & 0 & 2 & 4 & 0 \\ 3 & -1 & 0 & 3 & -1 \end{vmatrix}$$

$$= 0\vec{i} + 6\vec{j} + 1\vec{k} - (0\vec{i} + 2\vec{j} + 0\vec{k})$$

$$\vec{s} = 2\vec{i} + 6\vec{j} + 1\vec{k}$$

$$\vec{s} = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}, T(1, 3, 2)$$

$$\frac{x-1}{2} = \frac{y-3}{6} = \frac{z-2}{1}$$

14.6

$$A(1, 0, 2)$$

$$B(0, 2, 1)$$

$$C(-2, 4, 1)$$

$$\vec{r} = \vec{r}_A + U\vec{s}_1 + V\vec{s}_2$$

$$\vec{r} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + U \cdot \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + V \cdot \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$$

$$\vec{s}_1 = \vec{AB} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$$

$$\vec{s}_2 = \vec{BC} = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$$

no common vector direction
 hence su parallel
 (are not)

$$\vec{r} = \begin{bmatrix} 1-U-2V \\ -3U+4V \\ 2-U \end{bmatrix}$$

$$h) \quad 3x - 2y + z - 2 = 0$$

$$A(0, 0, 2)$$

$$B(0, -1, 0)$$

$$C\left(\frac{2}{3}, 0, 0\right) \dots \text{ista linija sadrži tačku } v)$$

Smisli 2 tačke u
prostoru 3.

e)

$$A(0, 2, 2)$$

$$\vec{n}_1 = \begin{matrix} s_1 \\ 2 = U \\ s_2 \\ -2U + 3V \end{matrix}$$

$$\vec{n}_2 = \begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix} + U \cdot \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + V \cdot \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -U \\ -2+U \\ 2-2U+3V \end{bmatrix}$$

paralelne ravni
imaju iste ili
proporcionalne vekt.
smjera \vec{s}_1 i \vec{s}_2

$$\vec{s}_1 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$$

$$\vec{s}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$



14.7

a)

$$A(x_0, y_0, z_0) \\ A(3, -1, 0)$$

$$B(-1, 0, 2)$$

$$C(2, 1, -1)$$

$$\vec{s}_1 = \vec{AB} = \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix}$$

$$\vec{s}_2 = \vec{CA} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\vec{n} = \vec{s}_1 \times \vec{s}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 1 & 2 \\ 1 & -2 & 1 \end{vmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

$$A \cdot (x - x_0) + B \cdot (y - y_0) + C \cdot (z - z_0)$$

$$5 \cdot (x - 3) + 6 \cdot (y + 1) + 7 \cdot (z - 0)$$

$$5x - 15 + 6y + 6 + 7z = 0$$

$$5x + 6y + 7z - 9 = 0$$

2)

$$\vec{n} = \begin{bmatrix} 1-v \\ 2+u+v \\ 3 \end{bmatrix}$$

$$T(1, 2, 3)$$

$$\vec{s}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{s}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{s}_1 \times \vec{s}_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

istea este
produsul
vectorial.

14.8

$$2x - y + z = 0 \quad \text{și} \quad x + 2y - z + 2 = 0$$

Gauss

$$\begin{bmatrix} 2 & -1 & 1 & | & 0 \\ 1 & 2 & -1 & | & -2 \end{bmatrix} \leftarrow \begin{matrix} \text{schimb} \\ \cdot (-2) \end{matrix}$$

iff je delecta: $\begin{bmatrix} 1 & 0 & \dots \\ 0 & 1 & \dots \end{bmatrix}$

iff $\begin{bmatrix} 0 & 1 & \dots \\ 1 & 0 & \dots \end{bmatrix}$

$$\begin{bmatrix} 0 & -5 & 3 & | & 4 \\ 1 & 2 & -1 & | & -2 \end{bmatrix} \leftarrow \begin{matrix} \cdot (-5) \\ \cdot (-2) \end{matrix}$$

$$\begin{bmatrix} 0 & 1 & -\frac{3}{5} & | & -\frac{4}{5} \\ 1 & 2 & -1 & | & -2 \end{bmatrix} \leftarrow \begin{matrix} \cdot (-2) \\ \cdot (-2) \end{matrix}$$

$$\begin{bmatrix} 0 & 1 & -\frac{3}{5} & | & -\frac{4}{5} \\ 1 & 0 & \frac{1}{5} & | & -\frac{2}{5} \end{bmatrix}$$

$$y - \frac{3}{5}z = -\frac{4}{5}$$

$$x + \frac{1}{5}z = -\frac{2}{5}$$

$$x = -\frac{2}{5} - \frac{1}{5}z$$

$$y = -\frac{4}{5} + \frac{3}{5}z$$

$$z = t$$

vectorial delecta:

$$\vec{n} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} 2 & -\frac{1}{5}t \\ 0 & \frac{1}{5}t \\ -4 & \frac{3}{5}t \\ t \end{bmatrix}$$

parametric delecta:

$$\frac{x + \frac{2}{5}}{-\frac{1}{5}} = \frac{y + \frac{4}{5}}{\frac{3}{5}} = \frac{z}{1}$$

b) \vec{n}_1 i \vec{n}_2 zapisati u vjecn
 obliku (kao u 7.a) i 7.b)
 i onda dalje isto kao 8.a)

14.9.

a) $x - 2y - z + 2 = 0$

$\frac{x-2}{-3} = \frac{y+1}{2} = \frac{z}{-1}$

$\vec{n} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$ $\vec{s} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

*međusobno
normalne*

\vec{s} i \vec{n} paralelni? nisu
 \vec{s} i \vec{n} sekudni? nisu

~~ne su paralelni~~

znaci da pravci i
 ravnina nisu okomit
 (dake imajete je skruker)

\vec{s} i \vec{n} okomiti?
 $\vec{s} \cdot \vec{n} = 0$?

$1 \cdot 3 - 2 \cdot 2 - 1 \cdot (-1) = 0$

pravac i ravnina su perpendikularni

b)

$\vec{s} = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$

$\vec{n} = \vec{s}_1 \times \vec{s}_2$ (pravac normalan ($\vec{n} = \vec{s}_1 \times \vec{s}_2$))
 i onda dalje isto kao 9.a)

14.40

$$\vec{n}_1 = \begin{bmatrix} 1-u \\ 2+u+2v \\ 3-v \end{bmatrix}$$

$$\vec{n}_2 = \begin{bmatrix} 1+2x \\ 0 \\ -1+x \end{bmatrix}$$

$$2x - y + 3z = 0$$

$$\vec{n} = \begin{bmatrix} 2-x \\ x \\ 1 \end{bmatrix}$$

$$2 \cdot (2-x) - x + 3 \cdot 1 + 2 = 0$$

$$4 - 2x - x + 3 + 2 = 0$$

$$-3x = -9 \quad | :(-3)$$

$$\boxed{x=3}$$

nasstina A u \vec{n}

$$2 - 3 = -1$$

③

①

$$T(-1, 3, 1)$$

2) ravninu u općem obliku
pravce u vektorski
i dalje istu kao u)

14.41

$$\vec{n}_1 = \begin{bmatrix} 1-u \\ 2+u+x \\ 3-v \end{bmatrix}$$

$$\vec{n}_2 = \begin{bmatrix} 1-x \\ 0 \\ -1+x \end{bmatrix}$$

$$\vec{s}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{s}_2 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$

$$\vec{n} = \vec{s}_1 \times \vec{s}_2 = \dots$$

$$\vec{n} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} \quad A(1, 2, 3)$$

$$A(x-x_0) + B(y-y_0) + C(z-z_0)$$

$$-x - y - 2z \stackrel{0}{=} 0$$

masi norden 0

$$T(1, 0, -1)$$

vduljerst masi

$$i \quad -x - y - 2z \stackrel{0}{=} 0$$

$$i \quad T(1, 0, -1)$$

Ko yo zo

Zadaci Koodle

$$\vec{r} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} \cos(60^\circ) & -\sin(60^\circ) \\ \sin(60^\circ) & \cos(60^\circ) \end{bmatrix} = \begin{bmatrix} -1.232050808 \\ 1.866025404 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1.232050808 \\ 1.866025404 \end{bmatrix} = \begin{bmatrix} -3.696152424 \\ 1.866025404 \end{bmatrix}$$

lasuljnye glanctnye masi

1.

$$T(-2, -3)$$

$$a) \quad \vec{x} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$



$$T'(-1, -1)$$

$$b) \quad T(-2, -3)$$

$$A(2, 1)$$

$$T(-2, -3) \quad \vec{r}_T = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 3 \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

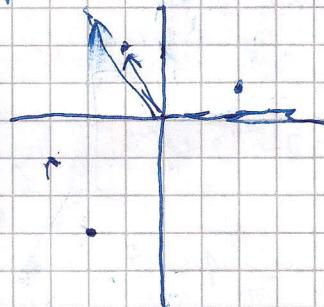
$$-2 + x = 2$$

$$x = 4$$

$$3 + y = 1$$

$$y = -4$$

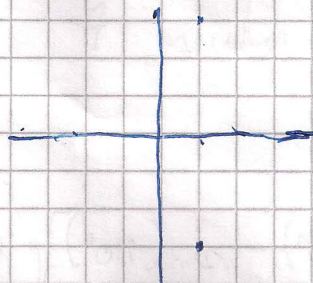
$$\begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$$



2.

$$T(1, -3)$$

$$a) x = 0$$



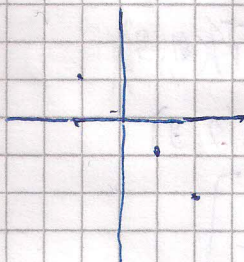
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

a)

$$y = -x$$

$$\begin{bmatrix} \end{bmatrix}$$



b)

$$T'(1, -3) \quad T(1, -3)$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

3.

$$T(5, 2)$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$T(2, 3)$$

$$\begin{bmatrix} h & 0 \\ 0 & h \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 5h \\ 2h \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$5h = 2 \quad | :5$$

$$h = \frac{2}{5}$$

$$h = \frac{3}{2}$$

4.

$$T(2,4)$$

$$\vec{r} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos(-180^\circ) & -\sin(-180^\circ) \\ \sin(-180^\circ) & \cos(-180^\circ) \end{bmatrix}$$

~~$$\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$~~

$$\cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

5.

$$a) \vec{r} = x$$

$$\begin{bmatrix} x \\ x \end{bmatrix}$$

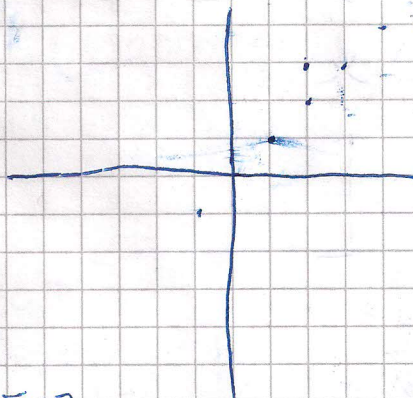
~~...~~

$$T(2,3)$$

$$\begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+x \\ 3+x \end{bmatrix}$$

b)

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x \end{bmatrix} = \begin{bmatrix} x \\ x \end{bmatrix}$$



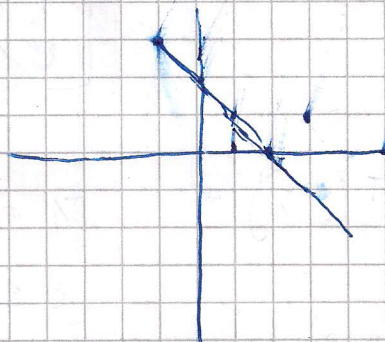
6.

$$y = 2 - x$$

$$\begin{bmatrix} x \\ 2-x \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3+x \\ 3-x \end{bmatrix}$$

$$x+2 = 2-x$$

$$y = -x$$



4. ogledni primjer 4. sklopa

$$y = x^3$$

$$\begin{bmatrix} x \\ x^3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x^3 \end{bmatrix} = \begin{bmatrix} 2x \\ x^3 \end{bmatrix}$$

A(1, -2)

$$\begin{bmatrix} -0.94 & -0.34 \end{bmatrix}$$

$$\begin{bmatrix} -0.9396926208 & -0.3420201433 \\ -0.3420201433 & 0.9396926208 \end{bmatrix}$$

- 0.255

$$\begin{bmatrix} -1-x \\ -4+2x \\ 1-3x \end{bmatrix} = \begin{bmatrix} 1-x \\ -4+x \\ 1-x \end{bmatrix}$$

$$-1-x = 1-u \rightarrow x = -u$$

$$-4+2x = -4+u$$

$$1-3x = 1-u$$

$$x = u$$

$$-x = 2-u$$

$$x = -2+u$$

$$\sqrt{-3x = -u /: (-3)}$$

$$x = \frac{u}{3}$$

~~$$x = u$$~~

~~$$-1 - \frac{u}{3} = 1 - u$$~~

0.7071067812

2.121320314

-0.7071067812

2.121320314

$$\begin{bmatrix} -x \\ -1+3x \\ -4+2x \end{bmatrix} \begin{bmatrix} -2+3x \\ 2x \\ -3+x \end{bmatrix}$$

$$-x = -2 + 3x$$

$$-1 + 3x = 2x \rightarrow 3x = 2x + 1$$

$$-4 + 2x = -3 + x$$

$$x = \frac{2x+1}{3}$$

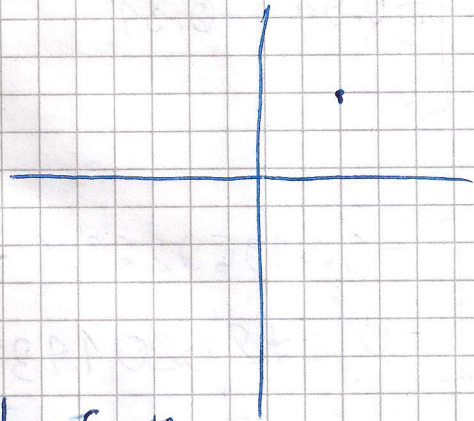
-1.532088886

-1.285575219

0.6427876097

0.766044431

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



$$\begin{bmatrix} -4-x \\ x \\ 2+2x \end{bmatrix} \begin{bmatrix} 1-x \\ -5+x \\ -2 \end{bmatrix}$$

$$x = -5 + x$$

$$2 + 2x = -2$$

$$-2 = -5 + x$$

$$x = 3$$

$$2x = -4$$

$$x = -2$$

$$\begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$x = -4$$

$$y = -1$$

$$z = 2$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

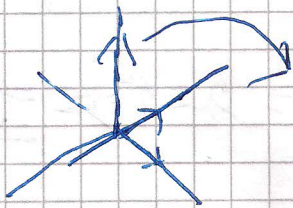
Ładunek sustaru

Kodur składowe i rotacje
pru unijel składowe ju
rotacyjne, dle S.P.

Kod se trzeci da su prawi gredni $\vec{S}_1 = k \vec{S}_2$

Kod se trzeci da su prawi dcaute

$$\vec{S}_1 \cdot \vec{S}_2 = 0$$



Kodu trzebnu wskedite da
prawi ludo dcaute su dca
prawi ordu $\vec{S}_1 \times \vec{S}_2 = \underline{\underline{S}}$

Výsledky za test

Yours

$$\left[\begin{array}{ccc|c} 5 & -1 & -1 & 2 \\ 3 & -1 & 1 & 4 \\ 1 & 2 & -1 & 1 \end{array} \right]$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & -1 & 1 & 4 \\ 5 & -1 & -1 & 2 \end{array} \right) \begin{array}{l} /(-3) \\ /(-5) \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -7 & 4 & 1 \\ 0 & -11 & 4 & -3 \end{array} \right] /:(-7)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{4}{7} & -\frac{1}{7} \\ 0 & -11 & 4 & -3 \end{array} \right] \begin{array}{l} / \cdot (-2) \\ / \cdot 11 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{7} & \frac{9}{7} \\ 0 & 1 & -\frac{4}{7} & -\frac{1}{7} \\ 0 & 0 & -\frac{16}{7} & -\frac{32}{7} \end{array} \right] /:(-\frac{16}{7})$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{7} & \frac{9}{7} \\ 0 & 1 & -\frac{4}{7} & -\frac{1}{7} \\ 0 & 0 & 1 & 2 \end{array} \right] \begin{array}{l} / \cdot (\frac{4}{7}) \\ / \cdot (-\frac{1}{7}) \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$x = 1$$

$$y = 1$$

$$z = 2$$

$$\left| \begin{array}{ccc|cc} 1 & 2 & 3 & 4 & 2 \\ 2 & 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 3 & 1 \end{array} \right|$$

$$= 2 + 6 - (8 + 0) = 8 - 17 = -9$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 1 \\ 3 & 1 & 3 & 1 & 1 \\ 1 & 3 & 1 & 3 & 1 \\ 2 & 1 & 2 & 1 & 1 \end{array} \right] \begin{array}{l} / \cdot (-3) \\ / \cdot (-1) \\ / \cdot (-2) \end{array} \left[\begin{array}{cccc|c} 1 & -2 & -1 & -2 & 1 \\ 0 & -5 & 0 & -5 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & -3 & 0 & -3 & 1 \end{array} \right]$$

$$1 \cdot \left[\begin{array}{cccc|c} -3 & 0 & -5 & -5 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 3 & 0 & -3 & -3 & 0 \end{array} \right] = 0 = 0$$

$$A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$c) \quad B \cdot A - A \cdot X = I$$

$$\rightarrow A^{-1} \cdot (-A \cdot X) = I - B \cdot A$$

$$X = A^{-1} \cdot I - B \cdot A \quad \begin{bmatrix} \frac{3}{2} & -1 \\ 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$

~~$$X = \frac{1}{2} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{2} & -1 \\ 0.5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$~~

~~$$X = \frac{1}{2} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{2} & -1 \\ 0.5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$~~

~~$$X = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$~~

$$X = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 1 \\ 1 & -4.5 \end{bmatrix}$$

$$E(7, -2) \quad D(6, 2) \quad B(-1, -1)$$

$$\vec{QE} = \vec{B} - \vec{BD}$$

~~$$Q = E - B$$~~

~~$$Q = E - B$$~~

$$2 \cdot (-5, 1) - (7, 3)$$

$$(-10, 2) - (7, 3)$$

$$(-17, -1)$$

1. 2 beder

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$2 \cdot \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} + \frac{1}{2}X = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\frac{1}{2}X = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix}$$

$$\frac{1}{2}X = \begin{bmatrix} 2 & -3 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix}$$

$$\frac{1}{2}X = \begin{bmatrix} 0 & -7 \\ 2 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & -14 \\ 4 & 8 \end{bmatrix}$$

2.

$$\begin{array}{l} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & 1 & 0 & -3 \\ -1 & -3 & 1 & -4 \end{array} \right] \begin{array}{l} / \cdot (-2) \\ / \cdot 1 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 5 & -2 & -1 \\ 0 & -5 & 2 & -5 \end{array} \right] \end{array}$$

$$\begin{array}{l} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & -5 & 2 & -5 \end{array} \right] \begin{array}{l} / \cdot 2 \\ / \cdot 5 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & -\frac{7}{5} \\ 0 & 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 0 & 6 \end{array} \right] \end{array}$$

4, 5 boden

$$x + \frac{1}{5}z = -\frac{7}{5}$$

$$x = -\frac{7}{5} - \frac{1}{5}z$$

$$\frac{1}{5}z = -x - \frac{7}{5} \quad | \cdot \frac{1}{5}$$

$$y - \frac{2}{5}z = -\frac{1}{5}$$

$$y = \frac{2}{5}z - \frac{1}{5}$$

$$z = -\frac{1}{5}x - 7$$

$$-\frac{2}{5}z = \frac{1}{5} - y \quad | \cdot (-\frac{5}{2})$$

$$z = -0.5 + \frac{2}{5}y$$

3. 3 boden

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} | \cdot (-2) \\ \leftarrow \\ \leftarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 1 & 2 & -2 & 0 & 1 \end{array} \right] \begin{array}{l} | \cdot (-1) \\ \leftarrow \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 & -1 & 1 \end{array} \right] \begin{array}{l} | \cdot (-2) \\ \leftarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} \end{array} \right] \begin{array}{l} \leftarrow \\ \leftarrow \\ | \cdot (-4) / \cdot 1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -2 & -1 & 2 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} \end{array} \right] = A^{-1}$$

4. 3 boden

$$\left[\begin{array}{cccc} 2 & 1 & 3 & 0 \\ 0 & -1 & -1 & 1 \\ 2 & 1 & -2 & 0 \\ 1 & -2 & 1 & 2 \end{array} \right] \begin{array}{l} | \cdot (-2) \\ | \cdot (-2) \\ \leftarrow \\ \leftarrow \end{array} \left[\begin{array}{cccc} 2 & -1 & 3 & 0 \\ 0 & -1 & -1 & 1 \\ 2 & 1 & -2 & 0 \\ 1 & 0 & 3 & 0 \end{array} \right] \begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \quad 1. \left[\begin{array}{ccc} 2 & 1 & 3 \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 2 & 1 & \\ 2 & 1 & -2 & 2 & 1 & \\ 1 & 0 & 3 & 1 & 0 & \end{array} \right] = 6 - 2 - (6 + 3) = -5$$

5. 0,5 Punkte

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$A \cdot X + B \cdot X = 2I$$

$$X \cdot (A + B) = 2I$$

$$X = 2 \cdot (A + B)^{-1}$$

$$X = 2 \cdot \left(\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \right)^{-1}$$

$$X = 2 \cdot \begin{pmatrix} 4 & -7 \\ 3 & 1 \end{pmatrix}^{-1}$$

$$X = \begin{bmatrix} \frac{2}{25} & \frac{14}{25} \\ \frac{6}{25} & \frac{8}{25} \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 4 & -7 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array} \right] \begin{array}{l} /: 4 \\ \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & -\frac{7}{4} & \frac{1}{4} & 0 \\ 0 & \frac{25}{4} & -\frac{3}{4} & 1 \end{array} \right] \begin{array}{l} / \cdot (-3) \\ /: \frac{25}{4} \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & -\frac{7}{4} & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{3}{25} & \frac{4}{25} \end{array} \right] \begin{array}{l} \leftarrow \\ /: \frac{7}{4} \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{25} & \frac{7}{25} \\ 0 & 1 & -\frac{3}{25} & \frac{4}{25} \end{array} \right]$$

6. 1 Punkt

a) $A(1, 0, -2)$
 $B(2, -1, 1)$
 $C(-2, 2, 0)$

$$\vec{BA} = (-1, 1, -3)$$

$$\vec{BC} = (-4, 3, -1)$$

b) $\vec{BA} - 2 \cdot \vec{BC}$

$$|(-1, 1, -3) - 2 \cdot (-4, 3, -1)|$$

$$|(-1, 1, -3) - (-8, 6, -2)|$$

$$|(7, -5, -1)|$$

c) 1 Punkt

$$\vec{BC} \cdot \vec{BA} = (-4) \cdot (-1) + 3 \cdot (-3) + (-1) \cdot (-3)$$

$$\vec{BC} \cdot \vec{BA} = 10$$