Geometrija u prostoru

13.1

a) $A(1,-2)$

$B(-3,1)$

$\vec{r} = \vec{r}_A + t\cdot\vec{s}$

$$\vec{r} = \begin{bmatrix}1\\-2\end{bmatrix} + t\cdot\begin{bmatrix}-4\\3\end{bmatrix} = \begin{bmatrix}1-4t\\-2+3t\end{bmatrix} \qquad t\in\mathbb{R}$$

$\vec{s} = \vec{AB} = -4\vec{i} + 3\vec{j}$ (dobije se od prošlog zad.)

b) $\underline{y = 2x - 1}$ → odaberemo bilo koji x, da dobijemo točku npr. $\underline{x=0}$

$\vec{s} = \begin{bmatrix}1\\2\end{bmatrix}$

$\vec{r} = \vec{r}_0 + t\cdot\vec{s}$ → $\begin{bmatrix}1\\2\end{bmatrix}$

$$\vec{r} = \begin{bmatrix}0\\-1\end{bmatrix} + t\cdot\begin{bmatrix}1\\2\end{bmatrix}$$

$y = 2\cdot 0 - 1$

$\underline{y = -1}$

$$\vec{r} = \begin{bmatrix}0 & t\\-1 & 2t\end{bmatrix} = \begin{bmatrix}t\\-1+2t\end{bmatrix}$$

$A(0,-1)$

$t\in\mathbb{R}$

c)

$A(3,1)$

$2y - x + 3 = 0$

$y = \frac{1}{2}x - \frac{3}{2}$

paralelni pravci imaju isti koeficijent smjera.

$k = \frac{1}{2} \rightarrow \vec{s} = \begin{bmatrix}1\\\frac{1}{2}\end{bmatrix}$

$\vec{r} = \vec{r}_A + t\cdot\vec{s}$

$$\vec{r} = \begin{bmatrix}3\\1\end{bmatrix} + t\begin{bmatrix}1\\\frac{1}{2}\end{bmatrix}$$

$$\vec{r} = \begin{bmatrix}3+t\\1+\frac{1}{2}t\end{bmatrix}$$

Ponavljanje za test

$$A=\begin{bmatrix} 2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ 2 & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & 1 & 2 & 1 \end{bmatrix} = 2\cdot(-1) = -2$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{array}\right] = 1+2-(4) = \underline{-1}$$

13.9

$y = x = f(x)$

$$\vec{r} = \begin{bmatrix} t \\ f(t) \end{bmatrix} \to \begin{bmatrix} t \\ t \end{bmatrix}$$

a) $\alpha = 90^\circ$ b) $\alpha = 60^\circ$ c) $\alpha = 40^\circ$

c) $$\vec{r} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \cdot \vec{r}$$

$$\vec{r} = \begin{bmatrix} \cos 40^\circ & -\sin 40^\circ \\ \sin 40^\circ & \cos 40^\circ \end{bmatrix} \cdot \begin{bmatrix} t \\ t \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 0.7660 & -0.6428 \\ 0.6428 & 0.7660 \end{bmatrix} \cdot \begin{bmatrix} t \\ t \end{bmatrix}$$

$$= \begin{bmatrix} 0.7660t - 0.6428t \\ 0.6428t + 0.7660t \end{bmatrix}$$

13.10

$\vec{r} = \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

a) $k_x = 2$

1. način

$\boxed{\begin{array}{l} x \text{ se mijenja} \\ y \text{ isti} \end{array}}$

$\vec{r}\,' = \begin{bmatrix} 2+2t \\ 2-t \end{bmatrix}$

2. način

$\vec{r}\,' = S_x \cdot \vec{r}$

$= \begin{bmatrix} k_x & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 2+2t \\ 2-t \end{bmatrix}$

b) $k_y = \frac{1}{3}$

1. način x isti

y se mijenja

$= \begin{bmatrix} 1+t \\ \frac{2}{3} - \frac{1}{3}t \end{bmatrix}$

2. način

$\vec{r}\,'' = S_y \cdot \vec{r}$

$\vec{r}\,'' = \begin{bmatrix} 1 & 0 \\ 0 & k_y \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

$= \begin{bmatrix} 1+t \\ \frac{2}{3} - \frac{1}{3}t \end{bmatrix}$

a)

$\vec{r} = \begin{bmatrix} 1+t \\ 2-t \end{bmatrix}$

npr. $t = 0$, $t = 1$

$1+0 = \boxed{1}$, $2-0 = \boxed{2}$ 1. točka $A(1, 2)$

$1+1 = \boxed{2}$, $2-1 = \boxed{1}$ 2. točka $B(2, 1)$

$A(2, 2)$ $B(4, 1)$

13.11

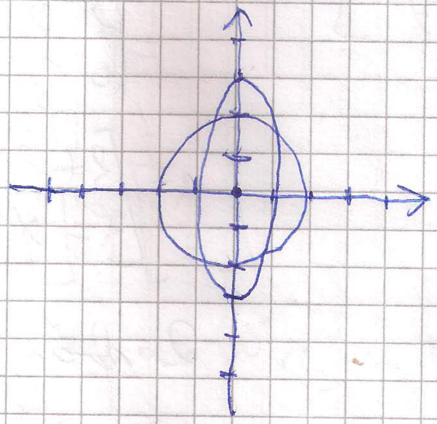
opěn: $\underline{(x-r)^2 + (y-q)^2 = R^2}$

$S(r, q)$

$S(0,0)$, $R = 2$

$$x^2 + y^2 = 4$$

vektorově: $\vec{r} = \begin{bmatrix} R\cos t \\ R\sin t \end{bmatrix} \rightarrow \begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{1} + \frac{y^2}{9} = 1$$

$a = 1$ $\quad b = \sqrt{9} = \underline{3}$

$\underline{kx}$

$2 \rightarrow 1$

$2 \cdot kx = 1 \;/ : 2$

$\boxed{kx = \frac{1}{2}}$

$\underline{ky}$

$2 \rightarrow 3$

$2 \cdot ky = 3 / 2$

$ky = \frac{3}{2} = \underline{1.}$

14.1

a) $A(1, -2, 2)$ $\quad \vec{s} = \vec{AB}$ $\quad$ $\underline{B \text{ minus } A}$

$B(0, -3, 1)$

$\vec{r} = \vec{r}_A + t \cdot \vec{s}$

$\vec{r} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + t \cdot \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

$\vec{r} = \begin{bmatrix} 1 - t \\ -2 - t \\ 2 - t \end{bmatrix}$

b)

$$\frac{x-2}{3} = \frac{y+1}{0} = \frac{z-0}{-1} \rightarrow \text{smjer}$$

$$\frac{x-x_A}{s_x} = \frac{y-y_A}{s_y} = \frac{z-z_A}{s_z}$$

$$\vec{s} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \; A(2,-1,0) \rightarrow \text{dobni podatke}$$

$$\vec{r} = \vec{r}_A + \lambda \cdot \vec{s}$$

$$\vec{r} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \lambda \cdot \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 2+3\lambda \\ -1 \\ -\lambda \end{bmatrix}, \; \lambda \in \mathbb{R}$$

c)

$A(3,-2,1)$

$$\vec{r} = \begin{bmatrix} 2-\lambda \\ 1+3\lambda \\ -2 \end{bmatrix}$$

posljednji parovi moraju biti jednaki ili proporcionalni vektoru smjera $\vec{s}$.

- neka su jednaki:

$$\vec{s} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$$\vec{r}\,' = \vec{r}_A + \lambda \cdot \vec{s}$$

$$\vec{r}\,' = \begin{vmatrix} 3 \\ -2 \\ 1 \end{vmatrix} + \lambda \cdot \begin{vmatrix} -1 \\ 3 \\ 0 \end{vmatrix}$$

$$\vec{r}\,' = \begin{bmatrix} 3-\lambda \\ -2+3\lambda \\ 1 \end{bmatrix}$$

14.2

a)

$A(3, -1, 0)$
$B(-1, -2, 2)$

$$\frac{x-3}{-4} = \frac{y+1}{-1} = \frac{z}{2}$$

$\vec{s} = \vec{AB}$

$\vec{s} = \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}$

b)

$\vec{r} = \begin{bmatrix} -t \\ 2 \\ 1-3t \end{bmatrix}$ → $A(0, 2, 1)$

1. način

$\vec{s} = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$

... opet ko gore

14.3

a)

$\vec{r}_1 = \begin{bmatrix} 2-3t \\ 2+t \\ 3 \end{bmatrix}$

izjednačim

$\vec{r}_2 = \begin{bmatrix} 4-4t \\ 4 \\ -1+2t \end{bmatrix}$

$$\begin{cases} 2-3t_1 = 4-4t_2 \\ 2+t_1 = 4 \\ 3 = -1+2t_2 \end{cases}$$

$2 + t_1 = 4$

$\boxed{t_1 = 2}$

$3 = -1 + 2t_2$

$t_2 = 2$

$$\begin{array}{ccc} & t_1 & & t_2 \\ 2-3\cdot & \boxed{2} & = 4-4\cdot & \boxed{2} \end{array}$$

$\boxed{-4 = -4}$ ✓

presjek
to je
u $(-4, 4$

uvrstim
t. t_2 u
t. t_1 u

$2 - 3 \cdot 2 = -$

b)

$$\frac{x}{3} = \frac{y-5}{-2} = \frac{z-2}{1}$$ $A_1(0, 5, 2)$ $\vec{s_1} = (3, -2, 1)$

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+1}{1}$$ $A_2(1, 2, -1)$ $\vec{s_2} = (2, 1, 1)$

$$\vec{r_1} = \begin{bmatrix} 3t \\ 5-2t \\ 2+1t \end{bmatrix} \quad i \quad \vec{r_2} \begin{bmatrix} 1+2t \\ 2+t \\ -1+t \end{bmatrix}$$

$$\begin{cases} 3t_1 = 1+2t_2 \\ 5-2t_1 = 2+t_2 \\ 2+t_1 = -1+t_2 \end{cases}$$

$2 + t_1 = -1 + t_2$

$t_1 = -3 + t_2$

$3t_1 = 1 + 2t_2$

$3 \cdot (-3 + t_2) = 1 + 2t_2$

$-9 + 3t_2 = 1 + 2t_2$

$t_2 = 10$

$t_1 = -3 + 10$

$t_1 = 7$

provjera

$5 - 2t_1 = 2 + t_2$

$5 - 2 \cdot 7 = 2 + 10$

$-9 = 12$

ne sijeku se

paralelni

$\vec{s_1} \neq \lambda \cdot \vec{s_2}$

mimoilazni su

14.4

a) $\vec{r_1} \begin{bmatrix} 2-t \\ 2+t \\ 3 \end{bmatrix}$ $\vec{r_2} = \begin{bmatrix} 4-\alpha t \\ 2t \\ -1+3\alpha t \end{bmatrix}$

okomiti su okomito: $\vec{s_1} \cdot \vec{s_2} = 0$

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -\alpha \\ 2 \\ 3\alpha \end{bmatrix} = 0$$

$-1 \cdot (-\alpha) + 1 \cdot 2 + (0 \cdot 3\alpha)$

$= 0$

$\alpha + 2 = 0$

$\alpha = -2$

13.2

a) $A(3,0)$

$B(-2,2)$

$\vec{r} = \vec{r_0} + t \cdot \vec{s}$

$$\vec{r} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 & -5t \\ 0 & +2t \end{bmatrix} = \begin{bmatrix} 3-5t \\ 2t \end{bmatrix}$$

$\vec{s} = \vec{AB} = -5\vec{i} + 2\vec{j}$ $\qquad t \in [0,1]$

b)

$A(2,-1)$

$B(0,4)$

$\vec{r} = \vec{r_0} + t \cdot \vec{s}$

$$\vec{r} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} + t \cdot \begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2-2t \\ -1+5t \end{bmatrix}$$

$t \in [0,1]$

$\vec{s} = \vec{AB} = -2\vec{i} + 5\vec{j}$

13.3

a) $\vec{r} = \begin{bmatrix} 1-2t \\ -3t \end{bmatrix}$

$t=0$

$A(1,0)$

$t=1$

$B(-1,-3)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$y - 0 = \frac{-3-0}{-1-1} \cdot (x-1)$$

$$y - 0 = \frac{-3}{-2} \cdot (x-1)$$

$$y - 0 = \frac{-3}{-2} x + \frac{3}{2}$$

$$y = \frac{-3}{-2} x + \frac{3}{2}$$

$$y = \frac{3}{2} x - \frac{3}{2}$$

b)

$$\vec{r} = \begin{bmatrix} 2 + t \\ 1 - 2t \end{bmatrix}$$

$t = 0$

$A(2, 1)$

$t = 1$

$B(3, -1)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$y - 1 = \frac{-1 - 1}{3 - 2} \cdot (x - 2)$$

$$y - 1 = \frac{-2}{1} \cdot (x - 2)$$

$$y - 1 = -2x + 4$$

$$y = -2x + 5$$

13.4

translacija

$t \in [0, 1]$

$$\vec{r}\,' = \vec{r} + \vec{t}$$

$$\vec{r} = \begin{bmatrix} 1 - 2t \\ 2 + t \end{bmatrix} \xrightarrow{t=0} A(1, 2)$$

$$\xrightarrow{t=1} B(-1, 3)$$

$$\vec{t} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$2 \rightarrow$ pomjera x-osu

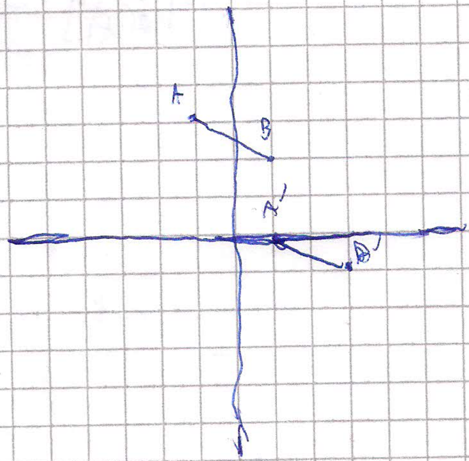
$-3 \rightarrow$ pomjera y-osu

$$\vec{r}\,' = \vec{r} + \vec{t}$$

$$\vec{r}\,' = \begin{bmatrix} 1 - 2t \\ 2 + t \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

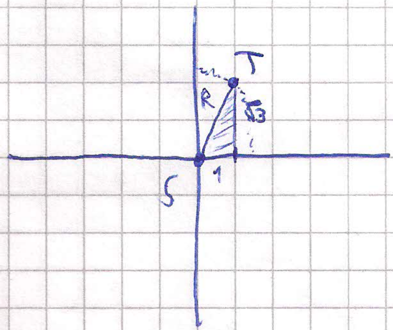
$$\vec{r}\,' = \begin{bmatrix} 3 - 2t \\ -1 + t \end{bmatrix} \xrightarrow{t=0} A'(3, -1)$$

$$\xrightarrow{t=1} B'(1, 0)$$



13.5

kružnica



$$\vec{r} = \begin{bmatrix} R\cos t \\ R\sin t \end{bmatrix}$$

$T(1, \sqrt{3})$

$$R^2 = 1^2 + (\sqrt{3})^2$$

$$R = 2$$

$$\vec{r} = \begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$$

$\vec{t} = \vec{ST}$ $\quad S(0,0)$, $T(1, \sqrt{3})$ –

$\vec{t} = 1\vec{i} + \sqrt{3}\,\vec{j}$

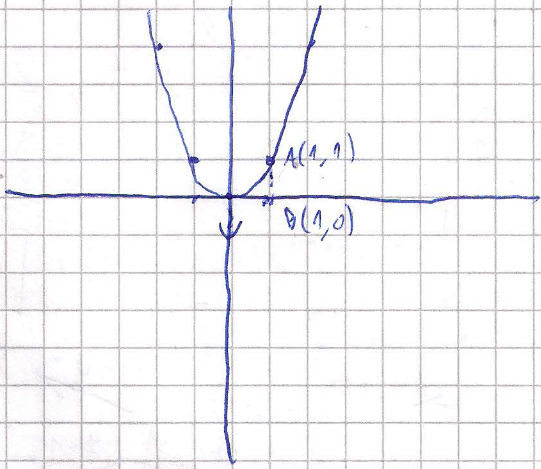
$$\vec{t} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

$$\vec{r}\,' = \begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix} + \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

$$\vec{r}\,' = \begin{bmatrix} 1 + 2\cos t \\ \sqrt{3} + 2\sin t \end{bmatrix}$$

13.6

$y = x^2 = f(x)$ $\qquad y = x^2$



$$\vec{r} = \begin{bmatrix} t \\ f(t) \end{bmatrix} = \begin{bmatrix} t \\ t^2 \end{bmatrix}$$

nultočke

$x_{1,2} = \ldots$

$x_{1,2} = 0.$

x	-2	-1	1	2
y	4	1	1	4

$$\vec{r}\,' = \begin{bmatrix} t \\ t^2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\vec{r}\,' = \begin{bmatrix} t \\ t^2 - 1 \end{bmatrix}$$

13.7

$$\vec{r} = \begin{bmatrix} -t \\ 3-2t \end{bmatrix}$$

preslikovanje preko:

1) x-osi : x isto
y mijenja predznak

2) y-osi :
x mijenja predznak
y isto

$$\vec{r}\,' = \begin{bmatrix} -t \\ -3+2t \end{bmatrix}$$

2. način

$$\vec{r}\,' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -t \\ 3-2t \end{bmatrix}$$

$$\vec{r}\,' = \begin{bmatrix} 1\cdot(-t)+0\cdot(3-2t) \\ 0\cdot(-t)-1\cdot(3-2t) \end{bmatrix}$$

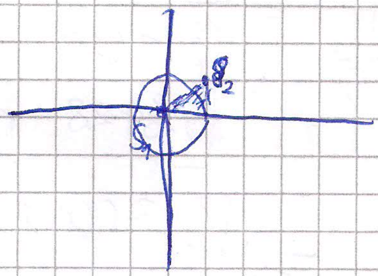
$$\vec{r}\,' = \begin{bmatrix} -t \\ -3+2t \end{bmatrix}$$

13.8

$S(1,1)$

$R=1$

$$\vec{r}_k = \begin{bmatrix} 1\cos t \\ 1\sin t \end{bmatrix} = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$



$$\vec{t} = \overrightarrow{S_1S_2}$$

$$\vec{t} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{r}\,' = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{r}\,' = \begin{bmatrix} 1+\cos t \\ 1+\sin t \end{bmatrix}$$

$$r_2' = \begin{bmatrix} -1-\cos t \\ 1+\sin t \end{bmatrix} \begin{matrix} \rightarrow \text{suprotni} \\ \rightarrow \text{isto} \end{matrix}$$ — nerazmjerno

b) paralelni

ili $\vec{s_1} = \vec{s_2}$

$\vec{s_1} = \underline{\lambda} \cdot \vec{s_2}$

neki broj

$\vec{s_1} = \begin{bmatrix}3\\-2\\1\end{bmatrix}$ $\vec{s_2} = \begin{bmatrix}-6\\2\alpha\\-2\end{bmatrix}$

s kojim brojem moram množiti da dobijem?

$\underline{\lambda = (-2)}$

$-2 \cdot \lambda = 2\alpha$

$-2 \cdot (-2) = 2\alpha$

$\boxed{\alpha = 2}$

Primjer ispita

Tablica sinusa i kosinusa nema!!! (U predlošku nema

1. $2A^T + \frac{1}{2}X = B \cdot C$

$\frac{1}{2}X = B \cdot C - 2A^T$

Samo riješiš i svugdje pomnožiš s 2 da se dobije X.

2.

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & 1 & 0 & -3 \\ -1 & -3 & 1 & -4 \end{array}\right] \begin{array}{l} / \cdot (-2) \quad / \cdot 1 \\ \\ \\ \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 5 & -2 & -1 \\ 0 & -5 & 2 & -5 \end{array}\right] / : 5$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & -\frac{2}{5} & -\frac{1}{5} \\ 0 & -5 & 2 & -5 \end{array}\right] \ldots$$

(Može se pojaviti jednadžba s parametrom (λ)) !!!

3.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

opet Gauss

4.

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & -1 & -1 & 1 \\ 2 & 1 & -2 & 0 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

U testu će uvijek biti 4x4 matricu, dakle Laplace razvoj uvijek.

5.

$$A \cdot \underline{X} + B \cdot \underline{X} = 2I$$

izlučivanje

$$(A+B)^{-1} \cdot (A+B) \cdot X = 2I$$

$$X = (A+B)^{-1} \cdot 2I$$

$$X = 2 \cdot (A+B)^{-1}$$

$$A+B = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}$$

$$A+B = \begin{vmatrix} 3 & -3 \\ 1 & 4 \end{vmatrix}$$

$$(A+B)^{-1} = \frac{1}{A+B} \quad \ldots$$

6.

a) $\overrightarrow{BA} = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$ ...

Koliki je kut između vektora? (Može doći kao pitanje tu)

c) i d)

jedan važni zbog 4. zadatka.

$\boxed{\text{4. je teži zad !!!}}$

$\boxed{\begin{array}{l}\text{2. Matrice} \\ \text{2. Gauss} \\ \text{1. Determinante} \\ \text{i Vektori ...}\end{array}}$

Zadaci na 3. zadatku

4. [illegible]

1.

2.

$R = 3$ $\quad S(0,0)$ prvo $\quad \begin{bmatrix} 3\cos t \\ 3\sin t \end{bmatrix}$ $\quad r' = r + t$

$$\begin{bmatrix} 1 + 3\cos t \\ -1 + 3\sin t \end{bmatrix}$$

3.* → ovo je teži primjer, dižu se pravi konkretni brojevi a kdk.

$$R = \begin{vmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{vmatrix} = \begin{vmatrix} \; \\ \text{''} \end{vmatrix} \cdot \begin{vmatrix} 1 + \sqrt{3}t \\ -1 + t \end{vmatrix} = \text{rješ}$$

$\vec{r}$: kružnica: postavi $\boxed{t}$ na neku vrijednost i dižanj od ishodišta i rotiraj.

4.

$y = x^3$

$$S = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} t \\ t^3 \end{bmatrix} = \begin{bmatrix} 2t \\ t^3 \end{bmatrix}$$

slika = original ali skalirano

Može doći i na testu da skaliranje ne bude direktno definirano nego mi moramo skužiti koliko trebamo skalirati.

5.

$$\vec{r} = \vec{r}_x + \vec{r}_y + \vec{r}_z \qquad \vec{r} = \begin{bmatrix} x & xv & + xu \\ y + yv + yu \\ z + zv + zu \end{bmatrix}$$

$\bar{v}$ smjer vektora...

6.

$$x - 2y - z + 3 = 0$$
$$-x + y + 2z - 1 = 0$$

→ Gauss

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ -1 & 1 & 2 & 1 \end{array}\right]$$

vektorski oblik $\vec{r} = \begin{bmatrix} 3t + 1 \\ t + 2 \\ t \end{bmatrix} \equiv \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

$$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z}{1}$$

kanonski oblik

7. → tr sjecište pravca s ravninom...

$$\vec{r} = \begin{vmatrix} 1 - t \\ 1 - t \\ 2t \end{vmatrix} \qquad -x + 2y - z + 2 = 0$$

$$\begin{cases} x = 1 - t \\ y = 1 - t \\ z = 2t \end{cases}$$

$$\boxed{t = \frac{1}{3}}$$

ovo se dobije za tri zapisa gore

4. [illegible]

1.

2.

$R = 3 \quad S(0,0)$ zato $\begin{bmatrix} 3\cos t \\ 3\sin t \end{bmatrix}$ $\quad r' = r + t$ $\begin{bmatrix} 1 + 3\cos t \\ -1 + 3\sin t \end{bmatrix}$

3.* → ovo je teži primjer, ali ću se pravi konkretni broje[illegible]

$$R = \begin{vmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{vmatrix} = \begin{vmatrix} 3 \text{ kolk.} \\ \text{„ " „} \end{vmatrix} \cdot \begin{vmatrix} 1 + \sqrt{3}t \\ -1 + t \end{vmatrix} = \text{rje}[illegible]$$

$\vec{r}$:   skica: postavi $\boxed{t}$ za neku vrijednost i dobivamo od osnovnog i rotiranog.

4.

$y = x^3$

$$S = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} t \\ t^3 \end{bmatrix} = \begin{bmatrix} 2t \\ t^3 \end{bmatrix}$$

skica = osnovni nas skalirana

Može doći i na testu da skaliranje ne bude direktno definirano nego mi moramo skužiti koliko treba skalirati.

5.

$\vec{r} = \vec{r}_x + \vec{r}_y + \vec{r}_z$ $\qquad \vec{r} = \begin{bmatrix} x & xV & +xU \\ y + yV + yU \\ z + zV + zU \end{bmatrix}$

zyleco nezavisi itp...

6.

$x - 2y - z + 3 = 0$
$-x + y + 2z - 1 = 0$ → Gauss

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ -1 & 1 & 2 & 1 \end{array}\right]$$

vektorski oblik $\vec{r} = \begin{bmatrix} 3t+1 \\ t+2 \\ t \end{bmatrix} \equiv \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

$$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z}{1}$$

kanonski oblik

7. → ta mjesta točke - stavit u ravninu pa

$\vec{r} = \begin{vmatrix} 1-t \\ 1-t \\ 2t \end{vmatrix}$ $\qquad -x + 2y - z + 2 = 0$

$$\begin{cases} x = 1 - t \\ y = 1 - t \\ z = 2t \end{cases}$$

$\boxed{t = \frac{1}{3}}$

ovo se dobije za tri zapisa gore

8. Iz jedne ravnine izvadimo neku točku

Dobit sjecište (uzet neki x, dobit y, z) ...

$x = 0 \qquad T(0,0,3)$

$y = 0$

$\boxed{z = 3}$ od formule

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

rješenje

14.4

c)
$$\vec{r}_1 = \begin{vmatrix} 1+2t \\ t \\ -1-t \end{vmatrix} \qquad \underbrace{\frac{x-3}{-1} = \frac{y-\alpha}{2} = \frac{z+1}{-1}}$$

$$\vec{r}_2 = \begin{vmatrix} 3-t \\ \alpha+2t \\ -1-t \end{vmatrix}$$

$$\begin{aligned} 1+2t &= 3-u \\ t &= \alpha + 2u \\ -1-t &= -1-u \end{aligned}$$

$\cancel{-1} - t = \cancel{-1} - u$

$-t = -u$

$\boxed{t = u}$

$1 + 2t = 3 - u$

$1 + 2u = 3 - u$

$$\boxed{\begin{aligned} u &= \frac{2}{3} \\ t &= \frac{2}{3} \end{aligned}}$$

$t = \alpha + 2u$

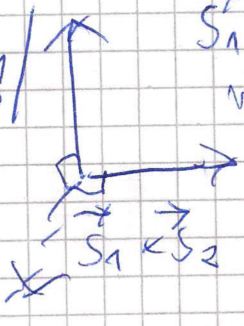
$\frac{2}{3} = \alpha + 2 \cdot \frac{2}{3}$

$\boxed{\alpha = -\frac{2}{3}}$

14.5
$$\vec{r}_1 = \begin{vmatrix} 2-t \\ 3 \\ 2t \end{vmatrix} \qquad \vec{r}_2 = \begin{vmatrix} 1+3t \\ 3-t \\ 2 \end{vmatrix}$$

$$\vec{s}_1 = \begin{vmatrix} -1 \\ 0 \\ 2 \end{vmatrix} \qquad \vec{s}_2 = \begin{vmatrix} 3 \\ -1 \\ 0 \end{vmatrix}$$

vektor okomit na oba vektora $\vec{s}_1$ i $\vec{s}_2$ je vektor $\vec{s}_1 \times \vec{s}_2$



$$\vec{s_1} \times \vec{s_2} = \left| \begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ -1 & 0 & 2 & -1 & 0 \\ 3 & -1 & 0 & 3 & -1 \end{array} \right|$$

$$= 0\vec{i} + 6\vec{j} + 1\vec{k} - (0\vec{k} - 2\vec{i} + 0\vec{j})$$

$$= 2\vec{i} + 6\vec{j} + 1\vec{k}$$

$$\vec{s} = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}, \quad T(1,3,2)$$

$$\frac{x-1}{2} = \frac{y-3}{6} = \frac{z-2}{1}$$

14.6

$A(1,0,2)$

$B(0,-3,1)$

$C(-2,1,1)$

$$\vec{r} = \vec{r_A} + u \cdot \vec{s_1} + v \cdot \vec{s_2}$$

$$\vec{r} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + u \cdot \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + v \cdot \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$$

$$\vec{s_1} = \vec{AB} = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$$

$$\vec{s_2} = \vec{BC} = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$$

ne smiješ uzeti vektore koji su paralelni (ovi nisu)

$$\vec{r} = \begin{bmatrix} 1 - u - 2v \\ -3u + 4v \\ 2 - u \end{bmatrix}$$

h) $3x - 2y + z - 2 = 0$

$A(0, 0, 2)$

$B(0, -1, 0)$

$C\left(\frac{2}{3}, 0, 0\right)$ ... isto kao zadatak v)

Izmisli 2 točke i izračunaj 3.

i)

$A(0, -2, 2)$

$$\vec{r}_1 = \begin{vmatrix} 2 \boxed{-u} \\ 1 + v \\ \boxed{-2u} + 3v \end{vmatrix}$$

paralelne ravnine imaju iste ili proporcionalne vektore smjera $\vec{s}_1$ i $\vec{s}_2$

$$\vec{r}_2 = \begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix} + u \cdot \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} + v \cdot \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -u \\ -2 + v \\ 2 - 2u + 3v \end{bmatrix}$$

$$\vec{s}_1 \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} \qquad \vec{s}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

14.7

a) $A(3, -1, 0)$ (x_0, y_0, z_0)

$B(-1, 0, 2)$

$C(2, 1, -1)$

$$\vec{s}_1 = \vec{AB} = \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix}$$

$$\vec{s}_2 = \vec{CA} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\vec{n} = \vec{s}_1 \times \vec{s}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 1 & 2 \\ 1 & -2 & 1 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -4 & 1 \\ 1 & -2 \end{matrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

$A \cdot (x - x_0) + B \cdot (y - y_0) + C \cdot (z - z_0)$

$5 \cdot (x - 3) + 6 \cdot (y + 1) + 7 \cdot (z - z_0)$

$5x - 15 + 6y + 6 + 7z = 0$

$$\boxed{5x + 6y + 7z - 9 = 0}$$

b)

$$\vec{r} = \begin{bmatrix} 1-v \\ 2+u+v \\ 3 \end{bmatrix} \qquad T(1,2,3)$$

$$\vec{s}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{s}_1 \times \vec{s}_2 = \left[\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 0 & 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\vec{s}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

...

isto kr pride od vodje.

14.8

$$2x - y + z = 0 \quad ; \quad x + 2y - z + 2 = 0$$

Gauss

cilj je dobiti: $\begin{bmatrix} 1 & 0 & \dots \\ 0 & 1 & \dots \end{bmatrix}$

ali $\begin{bmatrix} 0 & 1 & \dots \\ 1 & 0 & \dots \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 1 & 2 & -1 & -2 \end{array}\right] /\cdot(-2)$$

$$\left[\begin{array}{ccc|c} 0 & -5 & 3 & 4 \\ 1 & 2 & -1 & -2 \end{array}\right] /:(-5)$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -\frac{3}{5} & -\frac{4}{5} \\ 1 & 2 & -1 & -2 \end{array}\right] /\cdot(-2)$$

$$\underbrace{\left[\begin{array}{ccc|c} 0 & 1 & -\frac{3}{5} & -\frac{4}{5} \\ 1 & 0 & \frac{1}{5} & -\frac{2}{5} \end{array}\right]}$$

$$y - \frac{3}{5}z = -\frac{4}{5}$$

$$x + \frac{1}{5}z = -\frac{2}{5}$$

$$x = -\frac{2}{5} - \frac{1}{5}t$$

$$y = -\frac{4}{5} + \frac{3}{5}t$$

$$z = t$$

vektorski oblik:

$$\vec{r} = \begin{bmatrix} \frac{2}{5} - \frac{1}{5}t \\ -\frac{4}{5} + \frac{3}{5}t \\ t \end{bmatrix}$$

kanonski oblik:

$$\frac{x + \frac{2}{5}}{-\frac{1}{5}} = \frac{y + \frac{4}{5}}{\frac{3}{5}} = \frac{z}{1}$$

b)

$\vec{r_1}$ i $\vec{r_2}$ zapisati u općem obliku (kao u 7.a) i 7.b))

i onda dalje isto kao 8.a)

14.9.

a) $\underbrace{x - 2y - z + 2 = 0}$

$\underbrace{\frac{x-2}{3} = \frac{y+1}{2} = \frac{z}{-1}}$

$$\vec{n} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \qquad \vec{s} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

množenje

množenje

$\vec{s}$ i $\vec{n}$ paralelni? nisu

$\vec{s}$ i $\vec{n}$ jednaki? nisu

znači da pravac i ravnina nisu okomiti

(dakle moguće je okomiti)

$\vec{s}$ i $\vec{n}$ okomiti?

$\vec{s} \cdot \vec{n} = 0$?

$1 \cdot 3 - 2 \cdot 2 - 1 \cdot (-1) = 0$ ✓

pravac i ravnina su paralelni

b) $\vec{s} = \begin{bmatrix} -2 \\ \frac{3}{2} \\ -1 \end{bmatrix}$ $\vec{n} = ?$ (pomoću normale ($\vec{n} = \vec{s_1} \times \vec{s_2}$))

i onda dalje isto kao 9.a)

14.10

$$\cancel{\vec{r}_1 = \begin{bmatrix} 1-u \\ 2+u+2v \\ 3-v \end{bmatrix}}$$

$$\cancel{\vec{r}_2 = \begin{bmatrix} 1+2t \\ 0 \\ -1+t \end{bmatrix}}$$

$$2x - y + 3z + 2 = 0$$

$$\vec{r} = \begin{array}{c} \begin{bmatrix} 2-t \\ t \\ 1 \end{bmatrix} \end{array} \begin{array}{c} x \\ y \\ z \end{array}$$

$$2\cdot(2-t) - t + 3\cdot 1 + 2 = 0$$

$$4 - 2t - t + 3 + 2 = 0$$

$$-3t = -9 \,/ : (-3)$$

$$\boxed{t = 3}$$

uvrstim t u $\vec{r}$)

$$2 - 3 = -1$$

$$3$$

$$1$$

$$T(-1, 3, 1)$$

b) ravninu u opći oblik
pravac u vektorski
i dalje isto kao a)

14.11

$$\vec{r}_1 \begin{bmatrix} 1-u \\ 2+u+\cancel{2v} \\ 3-v \end{bmatrix}$$

$$\vec{r}_2 = \begin{bmatrix} 1-t \\ 0 \\ -1+t \end{bmatrix}$$

$$\vec{s}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{s}_2 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$

$$\vec{n} = \vec{s}_1 \times \vec{s}_2 = \ldots$$

$$\vec{n} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} \qquad A(1, 2, 3)$$

$$A\cdot(x - x_0) + B\cdot(y - y_0) + C\cdot(z - z_0)$$

$$-x - y - 2z \underbrace{+0}_{0} = 0$$

nasi vrednost

$T(1, 0, -1)$

Vdaljenost ravnine

i $-x - y - 2z + 0 = 0$

i $T(1, 0, -1)$

$x_0 \; y_0 \; z_0$

Zadeve Moodle

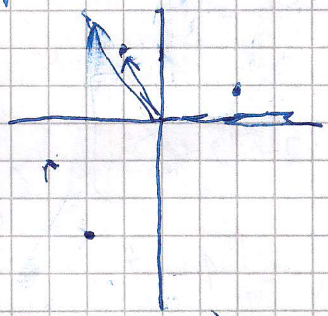
$$\vec{r} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} \cos(60^\circ) & -\sin(60^\circ) \\ \sin(60^\circ) & \cos(60^\circ) \end{bmatrix} = \begin{bmatrix} -1.232050808 \\ 1.866025404 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1.232050808 \\ 1.866025404 \end{bmatrix} = \begin{bmatrix} -3.696152424 \\ 1.866025404 \end{bmatrix}$$

Ponovitev geometrije ravnine

1.

$T(-2, -3)$



a) $\vec{x} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$T'(-3, -1)$

b) $T(-2, -3)$ $\quad A(2, 1)$

$$\begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 3 \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$-2 + x = 2$

$x = 4$

$3 + y = 1$

$y = 4$

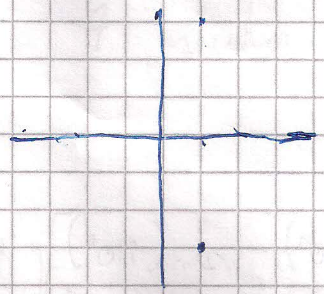
$T(-2, -3)$

c) $\vec{r}_T = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$$

2.

$T(1, -3)$



a) x - os

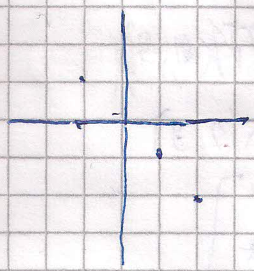
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

a)

$y = -x$

$\Big[$



c)

$T'(-1, -3) \quad T(1, -3)$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

3.

$T(5, 2)$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$T'(2, 3)$

$$\begin{bmatrix} k & 0 \\ 0 & l \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 5k \\ 2l \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$5k = 2 \quad / : 5$

$k = \frac{2}{5}$

$l = \frac{3}{2}$

4.

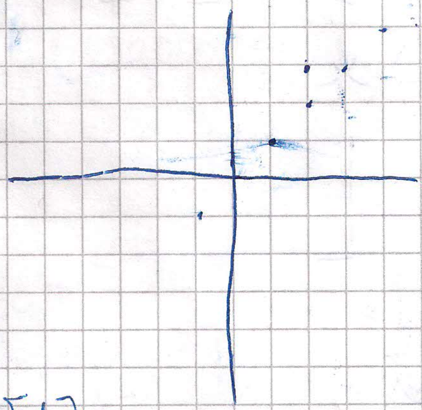
$T(2,4)$

$\vec{r} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $\qquad R = \begin{bmatrix} \cos(-180^\circ) & -\sin(-180^\circ) \\ \sin(-180^\circ) & \cos(-180^\circ) \end{bmatrix}$

$\cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$

5. a) $y = x$ $\qquad \begin{bmatrix} t \\ t \end{bmatrix}$

$T(2,3)$



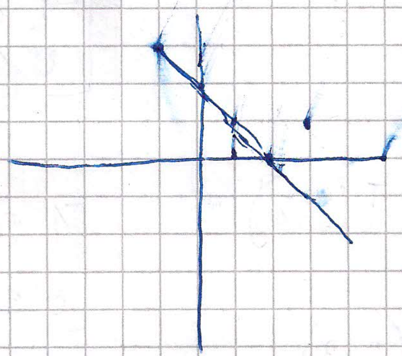
$\begin{bmatrix} t \\ t \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+t \\ 3+t \end{bmatrix}$

b)

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} t \\ t \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix}$

6.

$y = 2 - x$



$\begin{bmatrix} t \\ 2-t \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3+t \\ 3-t \end{bmatrix}$

$y+2 = 2-x$

$y = -x$

4. ogledni primjer 4. ishoda

$$y = x^3$$

$$\begin{bmatrix} t \\ t^3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} t \\ t^3 \end{bmatrix} = \begin{bmatrix} 2t \\ t^3 \end{bmatrix}$$

$A(1, -2)$

$$\begin{bmatrix} -0.94 & -0.34 \\ & \end{bmatrix}$$

$$\begin{bmatrix} -0.9396926208 & -0.3420201433 \\ -0.3420201433 & 0.9396926208 \end{bmatrix}$$

-0.255

$$\begin{bmatrix} -1-t \\ -4+2t \\ 1-3t \end{bmatrix} \quad i \quad \begin{bmatrix} 1-t \\ -4+t \\ 1-t \end{bmatrix}$$

$-1-t = 1-u \rightarrow \cancel{t = -u}$

$-4+2t = -4+u \qquad \cancel{t = u}$

$1-3t = 1-u \qquad -t = 2-u$

$t = -2+u$

$-3t = -u \,/ : (-3)$

$t = \frac{u}{3}$

$-1 - \frac{u}{3} = 1 - u$

0.7071067812 2.121320344

-0.7071067812 2.121320344

$$\begin{bmatrix} -t \\ -1+3t \\ -4+2t \end{bmatrix} \quad \begin{bmatrix} -2+3t \\ 2t \\ -3+t \end{bmatrix}$$

$-t = -2+3u$

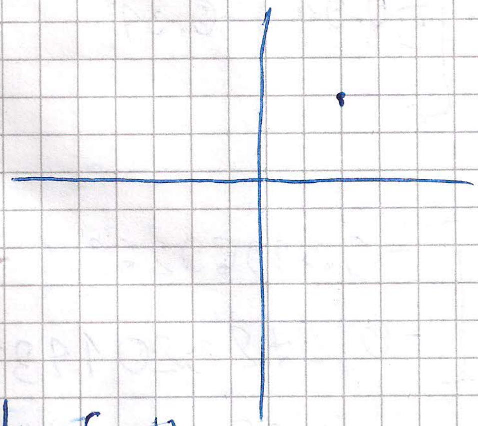
$-1+3t = 2u \rightarrow 3t = 2u+1$

$-4+2t = -3+u \qquad t = \frac{2u+1}{3}$

-1.532088886 -1.285575219

0.6427876097 0.7660444431

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix}$$



$$\begin{bmatrix} 4-t \\ t \\ 2+2t \end{bmatrix} \quad \begin{bmatrix} 1-t \\ -5+t \\ -2 \end{bmatrix}$$

$t = -5+u$

$2+2t = -2$

$2t = -4$

$t = -2$

$-2 = -5+u$

$u = 3$

$$\begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$x = -4$

$y = -1$

$z = 2$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

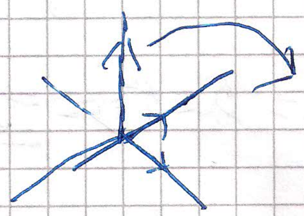
Sadatni sustav

Kod skaliranja i rotacije
prvo učinjeti skaliranje pa
rotiranje, dobile $\underline{S \cdot R}$

Kad se traži da su pravci paralelni $\vec{s}_1 = \underset{\substack{\downarrow \\ \text{realni broj}}}{k}\vec{s}_2$

Kad se traži da su pravci okomiti

$\underline{\vec{s}_1 \cdot \vec{s}_2 = 0}$



Kada trebamo odrediti da
pravac bude okomit na dva
pravca onda $\vec{s}_1 \times \vec{s}_2 = \underline{s}$

Vježba za test

Gauss

$$\left[\begin{array}{ccc|c} 5 & -1 & -1 & 2 \\ 3 & -1 & 1 & 4 \\ 1 & 2 & -1 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & -1 & 1 & 4 \\ 5 & -1 & -1 & 2 \end{array}\right] /\cdot(-3) \;\; /\cdot(-5)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -7 & 4 & 1 \\ 0 & -11 & 4 & -3 \end{array}\right] /:(-7) \qquad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{4}{7} & -\frac{1}{7} \\ 0 & -11 & 4 & -3 \end{array}\right] /\cdot(-2) \;\; /\cdot 11$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{7} & \frac{9}{7} \\ 0 & 1 & -\frac{4}{7} & -\frac{1}{7} \\ 0 & 0 & -\frac{16}{7} & -\frac{32}{7} \end{array}\right] /:\left(-\frac{16}{7}\right) \qquad \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{7} & \frac{9}{7} \\ 0 & 1 & -\frac{4}{7} & -\frac{1}{7} \\ 0 & 0 & 1 & 2 \end{array}\right] /\cdot\left(\frac{4}{7}\right) \;\; /\cdot\left(-\frac{1}{7}\right)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x = 1$

$y = 1$

$z = 2$

$$\left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 2 & 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 3 & 1 \end{array}\right| = 2 + 6 - (8 + 0) = 8 - 17 = \underline{-9}$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \\ 1 & 3 & 1 & 3 \\ 2 & 1 & 2 & 1 \end{array}\right] /\cdot(-3) \;\; /\cdot(-1) \;\; /\cdot(-2) \qquad \left[\begin{array}{cccc} +1 & -2 & +1 & -2 \\ 0 & -5 & 0 & -5 \\ 0 & 1 & 0 & 1 \\ 0 & -3 & 0 & -3 \end{array}\right]$$

$$1 \cdot \left[\begin{array}{ccc|cc} -5 & 0 & -5 & -5 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ -3 & 0 & -3 & -3 & 0 \end{array}\right] = 0 = 0$$

$$A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

c) $B \cdot A - A \cdot X = I$

$$-A^{-1} \,/\, -A \cdot X = I - B \cdot A$$

$$X = A^{-1} \cdot I - B \cdot A \qquad \begin{bmatrix} \frac{3}{2} & -1 \\ 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$

$$X = \frac{1}{2} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \qquad \begin{bmatrix} \frac{3}{2} & -1 \\ 0.5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 1 \\ 1 & -4.5 \end{bmatrix}$$

$E(7,-2) \quad D(6,2) \quad B(-1,-1)$

$$2\,\overrightarrow{EB} - \overrightarrow{BD}$$

$$2 \cdot (-5, 1) - (7, 3)$$

$$(-10, 2) - (7, 3)$$

$$(-17, -1)$$

1. 2 boda

$$A=\begin{bmatrix}1&-2\\2&1\end{bmatrix}\quad B=\begin{bmatrix}1&0&-1\\0&2&2\end{bmatrix}\quad C=\begin{bmatrix}2&-1\\-1&1\\0&2\end{bmatrix}$$

$$2\cdot\begin{bmatrix}1&2\\-2&1\end{bmatrix}+\frac{1}{2}X=\begin{bmatrix}1&0&-1\\0&2&2\end{bmatrix}\cdot\begin{bmatrix}2&-1\\-1&1\\0&2\end{bmatrix}$$

$$\frac{1}{2}x=\begin{bmatrix}1&0&-1\\0&2&2\end{bmatrix}\cdot\begin{bmatrix}2&-1\\-1&1\\0&2\end{bmatrix}-\begin{bmatrix}2&4\\-4&2\end{bmatrix}$$

$$\frac{1}{2}x=\begin{bmatrix}2&-3\\-2&6\end{bmatrix}-\begin{bmatrix}2&4\\-4&2\end{bmatrix}$$

$$\frac{1}{2}x=\begin{bmatrix}0&-7\\2&4\end{bmatrix}$$

$$x=\begin{bmatrix}0&-14\\4&8\end{bmatrix}$$

2.

$$\left[\begin{array}{ccc|c}1&-2&1&-1\\2&1&0&-3\\-1&-3&1&-4\end{array}\right]\ /\cdot(-2)\ /\cdot 1\quad\left[\begin{array}{ccc|c}1&-2&1&-1\\0&5&-2&-1\\0&-5&2&-5\end{array}\right]\ /:$$

$$\left[\begin{array}{ccc|c}1&-2&1&-1\\0&1&-\frac{2}{5}&-\frac{1}{5}\\0&-5&2&-5\end{array}\right]\ /\cdot 2\ /\cdot 5\quad\left[\begin{array}{ccc|c}1&0&\frac{1}{5}&-\frac{7}{5}\\0&1&-\frac{2}{5}&-\frac{1}{5}\\0&0&0&-6\end{array}\right]$$

1,5 bod

$$\cancel{x} + \frac{1}{5}z = -\frac{7}{5} \qquad x = -\frac{7}{5} - \frac{1}{5}z \qquad \frac{1}{5}z = -x - \frac{7}{5} \;/ : \frac{1}{5}$$

$$y - \frac{2}{5}z = -\frac{1}{5} \qquad y = \frac{2}{5}z - \frac{1}{5} \qquad z = -\frac{1}{5}x - 7$$

$$-\frac{2}{5}z = \frac{1}{5} - y \;/ : \left(-\frac{2}{5}\right)$$

$$z = -0.5 + \frac{2}{5}y$$

3. 3 body

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] / \cdot(-2) \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 1 & 2 & -2 & 0 & 1 \end{array}\right] / \cdot(-1)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 & -1 & 1 \end{array}\right] / : (-2) \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} \end{array}\right] / \cdot(-4) \; / \cdot 1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -2 & -1 & 2 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} \end{array}\right] = A^{-1}$$

4. 3 body

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & -1 & -1 & 1 \\ 2 & 1 & -2 & 0 \\ 1 & -2 & 1 & 2 \end{bmatrix} / \cdot(-2) \quad \begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & -1 & -1 & 1 \\ 2 & 1 & -2 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix} \quad 1. \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|cc} 2 & 1 & 3 & 2 & 1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 0 & 3 & 1 & 0 \end{array}\right] = 6 - 2 - (6 + 3) = \cancel{-8-0} = 4 - 9 = \boxed{-5}$$

0.5 boda

5.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$A \cdot X + B \cdot X = 2I$$

$$X \cdot (A \cdot B) = 2I$$

$$X = 2 \cdot (A \cdot B)^{-1}$$

$$X = 2 \cdot \left(\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \right)^{-1}$$

$$X = 2 \cdot \begin{pmatrix} 4 & -7 \\ 3 & 1 \end{pmatrix}^{-1}$$

$$X = \begin{bmatrix} \frac{2}{25} & \frac{11}{25} \\ -\frac{6}{25} & \frac{8}{25} \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 4 & -7 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right] / :4$$

$$\left[\begin{array}{cc|cc} 1 & -\frac{7}{4} & \frac{1}{4} & 0 \\ 0 & \frac{25}{4} & -\frac{3}{4} & 1 \end{array}\right] \begin{array}{l} / \cdot (-3) \\ / : \frac{25}{4} \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & -\frac{7}{4} & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{3}{25} & \frac{4}{25} \end{array}\right] / \cdot \frac{7}{4}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{25} & \frac{7}{25} \\ 0 & 1 & -\frac{3}{25} & \frac{4}{25} \end{array}\right]$$

6. [1]

a) $A(4, 0, -2)$

$B(2, -1, 1)$

$C(-2, 2, 0)$

$$\overrightarrow{BA} = (-1, 1, -3)$$

$$\overrightarrow{BC} = (-4, 3, -1)$$

b) [0]

$$\left| \overrightarrow{BA} - 2 \cdot \overrightarrow{BC} \right|$$

$$\left| (-1, 1, -3) - 2 \cdot (-4, 3, -1) \right|$$

$$\left| (-1, 1, -3) - (-8, 6, -2) \right|$$

$$\left| (7, -5, -1) \right|$$

1 bod

c)

$$\overrightarrow{BC} \cdot \overrightarrow{BA} = (-4) \cdot (-1) + 3 \cdot \ldots$$
$$(-3) \cdot (-\ldots$$

$$\overrightarrow{BC} \cdot \overrightarrow{BA} = 10$$